Late to Recessions: Stocks and the Business Cycle*

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Job Market Paper
This version: November 2018

Abstract

I show that the state of the business cycle is far more informative about expected stock returns than previously recognized. I identify business-cycle turning points by estimating a state-space model using real-time macroeconomic and financial data. I find that returns are predictably negative for the first 4-6 months after the onset of recessions, and only become high thereafter. Moreover, returns exhibit substantial momentum in recessions, whereas in expansions they display the mild reversals expected from discount rate changes. A market timing strategy that optimally exploits these returns’ business-cycle dependence produces a 60% increase in the buy-and-hold Sharpe ratio and substantially outperforms popular timing strategies in out-of-sample tests. In contrast with previous literature, the predictability is mostly due to the macro quantities. Using investor forecast surveys, I show that my findings are consistent with investors’ slow reaction to recessions.

*I am immensely grateful to my advisors Itamar Drechsler, Frank Schorfheide, Robert Stambaugh, and Amir Yaron (chair) for their invaluable guidance in writing this paper. I thank Michael Brennan, Anna Cororaton, Winston Dou, Joao Gomes, Deeksha Gupta, Lars Hansen, Mete Kilic, Jianan Liu, Felix Matthys, Christian Opp, Krishna Ramaswamy, Salvador Rivero, Nikolai Roussanov, Jeremy Siegel, Thomas Sargent, Sang Byung Seo, Dongho Song, Jessica Wachter, and seminar participants at ITAM, Panagora Asset Management, and The Wharton School. I am also thankful for financial support from the Rodney L. White Center for Financial Research.

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1 Introduction

A large and expanding literature explores the relation between stock returns and the business cycle. The traditional view is that stocks are rationally priced to immediately reflect investors’ expectations about future economic activity. In addition, the consensus is that expected excess returns on stocks (equivalently, risk premia) are positive, vary over time, and display strong counter-cyclical behavior. In this paper, I argue that stock prices are less efficient at incorporating fluctuations in the business cycle than what the literature has previously acknowledged. Specifically, I show that stock prices are late in reflecting information that signals the onset of recessions in an economically significant way. This then has important consequences for the behavior of expected returns.

Using real-time macroeconomic and financial data, I identify business-cycle turning points by developing and estimating out of sample a state-space model. The central finding is that expected returns are negative for several months following the onset of recessions, before rising to persistent higher-than-average levels. Further conditioning on the business cycle reveals a strong state-dependence in autocorrelations. That is, returns exhibit substantial momentum in recessions, whereas in expansions they display the mild reversals expected from discount rate changes. Using the model’s optimal forecast, I then construct a market timing strategy to analyze the implied return predictability and to assess the economic relevance of my findings. The strategy produces significant risk-adjusted returns. For the aggregate market, the strategy earns an annualized alpha of 7.4%, increases Sharpe ratios by 60% over the fully invested buy-and-hold approach, generates big utility gains for a mean-variance investor of around 33 monthly basis points, and produces a significant monthly out-of-sample $R^2$ statistic of 2%.

The central finding of the paper is exemplified in the right panel of Figure 1. An investor who buys stocks in the first 4 months of a recession –as defined by the probability of being in a recession (see the left panel of Figure 1)– and holds them for 6 months, earns an annualized average excess return of -8%. However, these negative returns are reversed in the following 12 months.


I assume that a recession occurs when the recession probability is above 50%. However, the 50% threshold rule is not essential. I obtain similar results with various threshold rules around 50%. The 6-month investment horizon is
Figure 1. Probability for a recessionary regime and 6-month equity excess returns.
The left panel plots the posterior median probability of being in a recessionary period. The light-gray bars indicate the NBER recession dates. Section 2.1.1 provides details for computing this probability. The right panel depicts the annualized 6-month realized excess market return during expected economic recessions. That is, suppose an investor buys stocks $m$ months after expecting to be in a recession and holds them for 6 months. The graph shows the excess returns for holding these stocks for this amount of time. The vertical bars represent 1 standard deviation error bars and account for heteroscedasticity and autocorrelation in the residuals up to six lags. For this plot, I assume that a recession starts whenever the probability first exceeds 50%. The sample runs from 1965:1 to 2016:12.

with persistently high excess market returns of around 13%. After a year, returns converge to the unconditional equity premium of 6.8%. From a market participant perspective, this pattern suggests that the investor can profit by selling at the start of recessions and by substantially increasing risk-taking several months later. From a risk-based perspective, the empirical pattern in Figure 1 presents a puzzle, because it is hard to explain negative average excess returns within any rational equilibrium model. Deepening the puzzle is the fact that these negative returns happen in precisely bad macroeconomic times, when risk-based theories generally suggest that compensations for risk should be, if anything, positive and high. As I will argue, the pattern instead appears more consistent with the idea that investors are late to act on macroeconomic news that signals the start of recessions.

My proposed state-space model links variations in expected returns to phases of the business cycle. The model relies on two main inputs. The first one filters business-cycle turning points from a large set of real-time macro and financial indicators. To avoid any look-ahead bias, I restrict the information set to those observations that were known and available to investors at each point in time. Thus, this dataset accounts for the fact that macro variables are sometimes released with a
delay or are significantly revised by the statistical agencies.

The second main input optimally exploits the state-dependent serial correlation in returns to predict future returns. The mild negative serial correlation in returns is consistent with a standard discount rate effect; shocks to expected returns tend to be accompanied by opposite shocks to current returns (French, Schwert, and Stambaugh [1987] Fama and French [1988]). The resultant mean-reversion offsets the positive serial correlation in returns coming from persistent and variable expected returns. In the data, the relative strength of these two offsetting channels is state-dependent: the negative channel dominates during expansions, whereas the positive one dominates during recessions. Hence, in expansions, returns exhibit a mild time-series reversal, and, thus, the filter negatively weighs recent past returns to predict future returns. Conversely, in recessions, returns exhibit time-series momentum, and, therefore, past returns are positively weighted when estimating the current expected return. However, the unconditional autocorrelation is small, given that it averages away this intriguing sign-flipping pattern.

To account for both the state of the business cycle and the state-dependent serial correlation in returns, I draw from the growing literature on density forecast combination. Specifically, my model dynamically combines the predictive densities from two individual models that differ in the amount in which expected returns vary (the relative strength of the positive serial correlation channel) using time-varying weights. These dynamic model weights correspond to the probability of being in an expansion or a recession. Building on the work of Hamilton [1989], my setup takes the form of a high-dimensional Markov-switching state-space model tailored toward monthly real-time data. A mixed-frequency approach and temporal aggregation enables me to sharpen inference about the regimes by incorporating the information contained in quarterly time-aggregated data and, at the same time, deals with variable reporting lags, early announcements, and missing observations of

3Formally, revisions in returns, $RN_{t+1}$, are due to revisions in the expectations about current and future dividend growth, $CFN_{t+1}$, or revisions in expected returns $DRN_{t+1}$, that is, $RN_{t+1} = CFN_{t+1} - DRN_{t+1}$. Hence, a negative shock to expected returns will generate a positive return shock, unless a striking negative dividend growth shock gets in the way. This generates negative serial correlation in ex-post returns. For returns to display momentum, a strong positive correlation between $CFN_{t+1}$ and $DRN_{t+1}$ is required. In Appendix A.10 I provide evidence for such a positive correlation via a VAR analysis. In the main text, I show that exploiting this strong momentum in returns produces substantial out-of-sample gains. These profits can be seen as its over-identifying restrictions.

each particular series happening throughout.

Using the model’s optimal estimate of expected returns, I analyze the economic relevance of my central finding via a market timing strategy. In line with Figure 1, my strategy generates a substantial part of its profits by reducing risk-taking for several months following the onset of a recession. At such times, my estimates of expected returns are negative—not positive and high. Shortly after having completely cashed out, my strategy returns to the market and increases risk-taking substantially. Put differently, the strategy produces profits in part by avoiding stock market crashes associated with recessions. The rest of the profits come from expansionary periods. During expansion, the strategy takes a contrarian position by exploiting mild mean-reversion in returns. I show that, although in the data mean-reversion is small, it is very important in a cumulative sense. The substantial gains of my market timing strategy may be surprising given the prevailing consensus in the extensive market timing literature that stock return predictability is highly unstable (e.g., Goyal and Welch 2008) and concentrated in short-lived predictability pockets (e.g., Farmer, Schmidt, and Timmermann 2018).

Therefore, to evaluate the robustness of my strategy, I conduct a wide range of tests. First, I show that identifying the state of the business cycle in a timely fashion is key for my results. The profitability significantly deteriorates if I use stale information of longer than 2 months in the forecasted regimes. My measure of business-cycle fluctuations relies on macro and financial variables. Importantly, my results suggest that macroeconomic variables play an important economic role, because omitting them considerably affects the strategy’s performance. In contrast, removing the information contained in the financial variables has only a modest effect on its overall performance. Second, I show that the outperformance of the strategy vanishes if I turn off the state-dependent autocorrelation in returns. Third, I show that the strategy can be further improved by also timing volatility, like in Moreira and Muir (2017). Finally, I extend the empirical analysis of my strategy in the appendix. Notably, the appendix shows that the strategy produces fairly large benefits even

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5Market timing strategies are also unpopular among practitioners. For example, Morgan Stanley—a leading investment bank—advised against market timing, calling it a “Losing Game” and stating that “trying to time market entries and exits rarely pays off and may, in fact, lead to investment losses”. See http://www.morganstanleyfa.com/public/projectfiles/459bbce6-c5a1-40e9-9ae7-f9d1a463a9f8.pdf. Similarly, Vanguard—a leading mutual fund company—also advised against market timing by positing that “time in the market is more important than timing the market”. See https://www.vanguardinvestor.co.uk/articles/latest-thoughts/investing-success/time-in-the-markets.
when subject to tight leverage/short-selling constraints and to realistic transaction costs. Moreover, I find remarkably similar profitability for returns on a wide range of characteristic-sorted portfolios, international equity markets, and across different sample periods. Given the profitability and robustness of the market timing strategy, I conclude that the model provides a good measure of expected returns over monthly horizons.

I analyze the economic implications of these findings in terms of the behavior of expected returns and examine the ability of leading rational and behavioral equilibrium models to explain the variation in expected returns documented in this paper. First, how can I reconcile my findings of making profit by reducing risk-taking at the onset of recessions with the long empirical evidence on countercyclical expected returns, which suggests, if anything, increasing risk-taking during these periods? I argue that although expected returns are negative at the onset of recessions, they subsequently rise to persistently positive and high levels (see Figure 1). Given that the former period is shorter than the latter, the overall annualized cumulative return is higher than the unconditional equity premium. That is, expected returns are higher in recessions. However, my evidence suggests selling during the initial phase of a recession and buying back in once the downward pressure on prices dissipates.

Second, how do the expected return dynamics filtered from the data compare with the ones implied by leading rational and behavioral asset pricing models? During expansions, I cannot determine whether the strategy’s profits are a result of rational variation in expected returns or due to market inefficiency. However, earning superior returns by reducing risk-taking during broadly defined bad expected times is difficult to explain in terms of risk. Instead, the evidence appears to be more consistent with an initial underreaction to news (Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999)). That is, news that signals the start of a recession is incorporated only slowly into prices, which induces the observed abnormal price drift during these times. As a result, my event-driven strategy earns significant returns by taking advantage of underreaction. I find support for this view in surveys of investor expectations.

Stock prices appear to drift after announcements or events related to earnings announcements (e.g., Bernard and Thomas (1989), Mendenhall (1991)), stock splits (e.g., Grinblatt, Masulis, and Titman (1984), Ikenberry, Rankine, and Stice (1996)), share repurchases (e.g., Ikenberry, Lakonishok, and Vermaelen (1995)), dividend initiations and omissions (e.g., Michael, Thaler, and Womack (1995)), seasoned equity offerings (e.g., Loughran and Ritter (1995); Spiess and Affleck-Graves (1995)), headline news (e.g., Chan (2003)), R&D expenses increase (e.g., Eberhart, Maxwell, and Siddique (2004)), the disposition effect (e.g., Frazzini (2006)), and analysts’ recommendations revisions (e.g., Bjerring, Lakonishok, and Vermaelen (1983)).
Although the variations in expected returns documented in this paper seem more consistent with behavioral than with risk-based asset pricing models, my findings also pose a challenge to behavioral theories. Current behavioral models do not explain all facts simultaneously. While models of underreaction to news are consistent with my findings during recessions, they cannot explain the behavior of expected returns in expansions. Conversely, models that incorporate extrapolative expectations (Barberis, Greenwood, Jin, and Shleifer (2015)) are consistent with my findings in expansions but inconsistent with my evidence in recessions. In other words, the challenge is to jointly explain why (1) expected returns are negative at the onset of recessions but positive and large shortly after and (2) a contrarian strategy during expansions earns substantial profits.

The paper is organized as follows. Section 2 introduces the model of expected returns. Section 3 presents the estimation results and describes how the model accounts for the state-dependent autocorrelation in returns. Section 4 measures the economic relevance of my findings via a market timing strategy. Section 5 analyzes the behavior of my expected returns estimates, discusses implications for general equilibrium models, and presents a survey-based explanation for my findings. Section 6 concludes and a detailed appendix extends my empirical analysis on many fronts.

2 Econometric framework

2.1 Model of expected returns

The model links the expected return and the business cycle and consists of two components. The first specifies the process for the business cycle (see Section 2.1.1), and the second component specifies the process for the return on stocks (see Section 2.1.2). The model accounts for these components via a dynamic prediction pool in which the pooling weights follow a Markov-switching process.

2.1.1 State of the business cycle. Business-cycle fluctuations are determined by analyzing the comovement of a broad range of macroeconomic and financial variables. Let \( y_{i,t+1} \) denote the \( i \)-th variable at time \( t + 1 \) and evolves as

\[
y_{i,t+1} = \gamma_i z_{t+1} + \epsilon_{i,t+1} \quad \text{for} \quad i = 1, \ldots, N,
\]  

(1)
where $z_{t+1}$ is the common component, whereas $e_{i,t+1}$ is the specific component. The parameter $\gamma_i$ measures the exposure of the $i$-th variable to the state variable $z_{t+1}$.

Based on the regime-switching model of Hamilton (1989), I assume that $z_{t+1}$ is generated by an AR process with Markov-switching deviations in the conditional mean, persistence, and volatility:

$$z_{t+1} = \mu_z(S_{t+1}) + \phi_z(S_{t+1})(z_t - \mu_z(S_t)) + \sigma_z(S_{t+1})\epsilon_{z,t+1},$$

with $\epsilon_{z,t+1} \sim \text{i.i.d} \mathcal{N}(0,1)$. $S_t$ is latent and denotes the economic regime indicator variable that signals whether the economy is in a contractionary or an expansionary period. The regime-switching conditional mean, $\mu_z(S_{t+1})$, and volatility, $\sigma^2_z(S_{t+1})$, attempt to capture transitions from periods of broad economic growth and low volatility, called expansions, to periods of broad economic contractions and high volatility, called recessions. For normalization purposes, I define $\mu_z(S_{t+1}) = \mu_0 + \mu_z S_{t+1}$, and impose the following restriction $\mu_z < 0$. Hence, if $S_{t+1} = 0$, I say that the economy is in a period of broad economic growth (i.e., $\mu_0 > \mu_0 + \mu_z$), whereas if $S_{t+1} = 1$, I say that the economy is in a contractionary period. Similarly, I define $\sigma^2_z(S_{t+1}) = \sigma^2_{z,0}(1 + h_{\sigma_z} S_{t+1})$ and impose $h_{\sigma_z} > -1$ to guarantee that the conditional volatility remains positive in the recessionary period. I further assume that transitions between regimes evolve according to a two-by-two transition matrix and that the specific component follows a simple AR(1) process

$$e_{i,t+1} = \mu_i + \psi_i e_{i,t} + \sigma_i \epsilon_{i,t+1},$$

with $\epsilon_{z,t+1} \sim \text{i.i.d} \mathcal{N}(0,1)$. The shocks $\epsilon_{z,t+1}$ and $e_{i,t+1}$ are independent of one another for all $t$ and $i$. Finally, the vector $\Theta_{bc}$ stacks the parameters associated with the process of the business cycle and specified by equations (1), (2), and (3). Hence, the subscript $bc$ in $\Theta_{bc}$.

**Filtered probability of a recession.** Let $p(S_t|\Theta_{bc}, \mathcal{F}_t)$ denote the posterior distribution of the state $S_t$ given the time-$t$ information set $\mathcal{F}_t$, which includes the macro and financial variables up to time $t$. Given $p(S_t|\Theta_{bc}, \mathcal{F}_t)$ we can compute the probability of being in a recession

$$\hat{\pi}_{t|t}(\Theta_{bc}) = E(S_t = 1|\Theta_{bc}, \mathcal{F}_t) = \int \mathbb{1}_{S_t=1} p(S_t|\Theta_{bc}, \mathcal{F}_t) dS_t,$$
where \(1_{S_t=j}\) is an indicator variable that equals one when regime \(j\) is in place. The estimate \(\pi_{t|t}(\Theta_{bc})\) represents the probability of being in the recessionary regime at time \(t\) conditional on the information set available at time \(t\) and on the parameter vector \(\Theta_{bc}\). I eliminate the dependence on the parameter vector \(\Theta_{bc}\) from the conditioning set in \(\pi_{t|t}(\Theta_{bc})\), by integrating out \(\Theta_{bc}\) using its posterior distribution \(p(\Theta_{bc}|F_t)\) as follows:

\[
\hat{\pi}_{t|t} = \int \hat{\pi}_{t|t}(\Theta_{bc}) p(\Theta_{bc}|F_t) d\Theta_{bc}.
\]

### 2.1.2 Stock returns.

Let \(r_{t+1}^e\) be the total return on the aggregate stock market in excess of the risk-free rate from time \(t\) to time \(t+1\):

\[
r_{t+1}^e = \mu_t + \epsilon_{r,t+1},
\]

where \(\mu_t\) denotes the unobservable conditional expected excess return (equivalently, risk premium or discount rate), whereas \(\epsilon_{r,t+1}\) is the unexpected excess return. The equity premium, \(\mu_t\), follows a first-order autoregressive process:

\[
\mu_{t+1} = \mu_0 + \rho (\mu_t - \mu_0) + \epsilon_{\mu,t+1},
\]

where \(\epsilon_{\mu,t+1}\) is the shock to expected returns, and \(\mu_0\) and \(\rho\) denote the unconditional mean and persistence of \(\mu_t\), respectively. The two shocks depend on the state of the business cycle and are conditionally Gaussian and uncorrelated at all leads and lags:

\[
(\epsilon_{r,t+1}, \epsilon_{\mu,t+1})' \sim iidN(0, \Sigma(S_t)) \quad \text{with} \quad \Sigma(S_t) = \begin{bmatrix} \sigma_{\epsilon r}^2(S_t) & \sigma_{\epsilon \mu}^2(S_t) \\ \sigma_{\epsilon \mu}^2(S_t) & \sigma_{\epsilon \mu}^2(S_t) \end{bmatrix}.
\]

The parameter \(\sigma_{\epsilon \mu}^2(S_t) = \rho_{\epsilon \mu, r}(S_t)\sigma_{\epsilon r}(S_t)\sigma_{\epsilon \mu}(S_t)\) measures the covariance between the two shocks, where \(\rho_{\epsilon \mu, r}(S_t) = \text{corr}_{S_t}(\epsilon_{r,t+1}, \epsilon_{\mu,t+1}).\) I further define the parameter \(\phi(S_t)\) as the state-dependent \(R^2\)-squared, which directly follows from the regime-switching nature of the variance-covariance matrix \(\Sigma(S_t)\):

\[
\phi(S_t) = \frac{\text{Var}_{S_t}(\mu_t)}{\text{Var}_{S_t}(r_{t}^e)}.
\]
In essence, the parameter $\phi(S_t)$ captures the idea that movements in the prices and quantities of aggregate risk (generating movements in expected returns) potentially vary across economic states. Finally, the vector $\Theta$ stacks all the parameters of the model $\left(\mu_0, \rho, \sigma^2_{\epsilon_r}(S_t), \sigma^2_{\epsilon_\mu}(S_t), \rho_{\epsilon_\lambda,r}(S_t)\right)$ for $S_t \in \{0, 1\}$.

**Filtered expected return.** Let $p(r_{t+1}^e|\Theta, r_{1:t}^e, S_t = j)$ denotes the predictive density obtained from equations (5), (6), and (7) by conditioning on a specific regime $j$ and the observed lagged returns up to time $t$, $r_{1:t}^e = \{r_1^e, \ldots, r_t^e\}$. Given $\hat{\pi}_{t|t}$, I can write the aggregate predictive density of returns as follows:

$$p(r_{t+1}^e|\Theta, r_{1:t}^e) = (1 - \hat{\pi}_{t|t}) p(r_{t+1}^e|\Theta, r_{1:t}^e, S_t = 0) + \hat{\pi}_{t|t} p(r_{t+1}^e|\Theta, r_{1:t}^e, S_t = 1),$$

which is a Markov-switching mixture of the individual predictive densities. Moreover, the forecasting weights assigned to each individual model are time varying and correspond to the probability of being in a period of broad macroeconomic contraction $\hat{\pi}_{t|t}$.

The filtered expected return is readily available from the aggregate predictive density in (9) by defining

$$\hat{r}_{t+1|t}(\Theta) = E(r_{t+1}^e|\Theta, r_{1:t}^e) = \int r_{t+1}^e p(r_{t+1}^e|\Theta, r_{1:t}^e) dr_{t+1}^e.$$  

Finally, in the empirical analysis, I drop the $\Theta$ argument by integrating out $\Theta$ from its posterior distribution, $p(\Theta|r_{1:t}^e)$, as follows:

$$E_t[r_{t+1}^e] = \hat{r}_{t+1|t} = \int \hat{r}_{t+1|t}(\Theta)p(\Theta|r_{1:t}^e)d\Theta,$$

where $E_t[r_{t+1}^e]$ denotes the marginal posterior mean of the filtered expected return. $E_t[r_{t+1}^e]$ is the key estimate of this paper and use it for the portfolio choice in Section 4.

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7In theory, stock returns are functions of state variables linked to the real economy. In turn, the real economy exhibits significant transitions from expansions to recessions. Thus, if the prices and quantities of aggregate risk are linked to the real economy, then movements in the real economy will generate time-varying expected returns (e.g., [Campbell and Cochrane 1999; Bansal and Yaron 2004]). If these prices and quantities vary across economic expansions and recessions, then stock return predictability, $\phi(S_t)$, will be tied to these business-cycle fluctuations.
2.2 Posterior inference

To conduct posterior inference, I write the likelihood function as follows:

$$p(r^c_{1:T} | \Theta, \pi_{1:T}) = \prod_{t=2}^{T} \{ (1 - \pi_t) p(r^c_t | \Theta, r^c_{1:t-1}, S_t = 0) + \pi_t p(r^c_t | \Theta, r^c_{1:t-1}, S_t = 1) \}. \tag{12}$$

Given the prior $p(\Theta)$ and the likelihood function in (12), Bayes’ theorem provides the posterior aggregate density function

$$p(\Theta | r^c_{1:T}, \pi_{1:T}) \propto p(r^c_{1:T} | \Theta, \pi_{1:T}) p(\Theta). \tag{13}$$

For posterior inference, I use a Bayesian Markov Chain Monte Carlo (MCMC) method\(^8\) Not all parameters are identified. However, identification can be achieved by slightly restricting the original parametrization. To focus the discussion on the identification problem and forecasting results, I relegate details on the state-space models, the estimation method, and regularity conditions to the appendix.

2.2.1 Identifying parameter restrictions. Appendix B.6 shows that the model is unidentified, because, for each regime, the matrix $\Sigma(S_t)$ has three free parameters and there are only two moments available to determine them. Hence, identification can be achieved by restricting any two elements of $\Sigma(S_t)$. A common choice in the literature is to impose an exclusion restriction\(^9\) which amounts to assuming that either $\sigma^2_{\varepsilon_{\mu}}(S_t) = 0$, $\sigma^2_{\varepsilon_r}(S_t) = 0$, or $\sigma^2_{\varepsilon_{\mu} \varepsilon_r}(S_t) = 0$. Rather than fixing (or excluding) any single element of $\Sigma(S_t)$, for which I do not have any strong prior views on what that value actually would be, I follow Aruoba, Diebold, Nalewaik, Schorfheide, and Song (2016) and impose a constraint on a linear combination of the variances $\sigma^2_{\varepsilon_{\mu}}(S_t)$ and $\sigma^2_{\varepsilon_r}(S_t)$, for which I have a more

\(^8\)The Bayesian approach has many advantages over maximum likelihood. For instance, it accounts for both parameter uncertainty and uncertainty from the latent states. Furthermore, it delivers finite-sample inference of various moments of interest by drawing from the posterior distribution.

\(^9\)Alternative econometric techniques that have been frequently employed to problems of this kind resort to identification through heteroskedasticity (e.g., Ehrmann, Fratzscher, and Rigobon 2011; Rigobon 2003) or sign restrictions. Both approaches are inappropriate for my purposes. With respect to the former, the main problem resides in that my setting does not satisfy the catch-up constraint in Proposition 2 of Rigobon (2003). (I would need to constrain the covariance between the shocks to a constant or to zero.) Intuitively, in my case, heteroskedasticity is not sufficient to achieve identification, because each regime adds two equations and three unknowns. Similarly, imposing sign restrictions is inappropriate in my setting, because this approach cannot uniquely pin down the parameters and hence leads to an extremely large admissible parameter space. Thus, it is essential to impose some sort of parameter restrictions.
natural prior belief. Specifically, I impose a degenerate prior on the conditional variance of the realized return \( r^e \):

\[
\text{Var}_{S_t}(r_{t+1}^e) = \sigma_t^2(S_t) = \sigma_r^2 (1 + h S_t)
\]  

(14)

by fixing \( \sigma_r^2 \) to some constant. The parameter \( \sigma_r^2 \) denotes the variance of \( r^e \) conditional on the expansionary regime (i.e., \( S_t = 0 \)). A data-based source of prior information about \( \sigma_r^2 \) could be to estimate the parameter for a sample period that precedes the benchmark forecasting estimation period\(^{10}\). In the empirical work, I will set \( \sigma_r^2 \) in this way, although I checked that the forecasting results are robust to a wide range of \( \sigma_r^2 \). Furthermore, I am allowing for a potentially different variance of \( r^e \) across regimes, because \( h \) is a free parameter to be estimated. For the second identifying restriction, I assume that the correlation parameter between the unexpected return and shocks to the expected return is stable across regimes (i.e., \( \rho_{\varepsilon_{t+1}, \mu, r}(S_t) = \rho_{\varepsilon_{t+1}, \mu, r} \) for \( S_t \in \{0, 1\} \)).

Overall, the matrix \( \Sigma(S_t) \) in (7) is given by:

\[
\Sigma(S_t) = \begin{bmatrix}
(1 - \rho^2) \phi(S_t) \sigma_t^2 (1 + h S_t) & \rho_{\varepsilon_{t+1}, \mu, r} \sqrt{(1 - \rho^2) \phi(S_t)(1 - \phi(S_t))} \sigma_t^2 (1 + h S_t) \\
(1 - \phi(S_t)) \sigma_t^2 (1 + h S_t)
\end{bmatrix},
\]

where I also reexpressed the model parameters associated with \( \Sigma(S_t) \) as a function of \( \phi(S_t) \) (the conditional \( R^2 \)-squared) to simplify and improve the clarity of the discussion that follows. Under these modifications, the parameter vector \( \Theta \) is now given by \((\mu_0, \rho, \phi(S_t), \rho_{\varepsilon_{t+1}, \mu, r}, h)\) for a fixed value of \( \sigma_r^2 \) and for \( S_t \in \{0, 1\} \).

3 Empirical implementation

This section describes the estimation results. In Section 3.1 I will present the real-time data set used. In Section 3.2.2 I will describe the filtered states, regime probabilities, and the parameter estimates. In Section 3.3 I will discuss how my measure of expected returns allows for state-dependent autocorrelation in returns. There, I also will explain how my strategy exploits this feature of the data and relate my strategy with common trend-following strategies used in the

\(^{10}\)To estimate \( \sigma_r^2 \), I need information about the regime indicator \( S_{t+1} \). To this end, I can use the ex post NBER recessionary index.
literature.

3.1 Real-time dataset

I am interested in forming a real-time estimate of the broad economic conditions at any given point in time in my sample. This objective leads to the following two challenges. First, I am limited on the type of observations that I can use to compute my estimate. Specifically, I can only use those variables that were available to investors at each point in time. The use of real-time data is crucial, because real-time data simulate the actual information set available to investors (e.g., Croushore and Stark 2001; Del Negro and Schorfheide 2013) and accounts for the fact that macro variables are sometimes released with a delay or receive substantial revisions by the statistical agencies.

Second, business-cycle fluctuations are not equal to movements in any single series. Instead, it is the common variation across many series in the economy. To this end, I selected 24 real-time macroeconomic and financial series that measure different parts of the economy and are available at the monthly or quarterly frequency. Specifically, I consider 13 real-time macro series that represent broad categories of the macroeconomy: real output and income (e.g., industrial production, gross domestic product), employment and hours (e.g., initial jobless claims), consumer spending (e.g., consumption expenditure), investment, and inflation (e.g., the Consumer Price Index). Furthermore, to sharpen inference about the economic states, I also consider information from 11 financial variables. The advantage of using forward-looking financial series is that they are observed in real time and contain negligible measurement errors (see Stock and Watson (2003) and the references therein). Specifically, the financial data set that I consider includes valuation ratios, the slope of the yield curve, credit and default spreads (e.g., TED, AAA-BAA), volatility measures (e.g., VIX), corporate issuing activity (e.g., net equity expansion), and debit balances (e.g., debit balances in the margin accounts of broker/dealers). All the series start at different periods, but all end in 2016:M12. The macro and financial indicators are the data counterpart of $y_{i,t}$ from Section 2.1.1 and I provide a detailed description of each in the appendix.

That macro time series receive substantial revisions could potentially affect my estimates of the regimes and give me an unfair advantage relative to a real-time investor. For example, for different vintages, the growth rates of consumption change nearly 5 percentage points (quarterly, at annual rates).
Figure 2. Filtered common component.
Panel (a) shows the filtered common component $z_t$. I also depict the year-on-year change in the Conference Board’s Economic Coincident Index published by the Federal Reserve Bank of Philadelphia. The Coincident Index includes four indicators: nonfarm payroll employment, the unemployment rate, average hours worked in manufacturing and wages and salaries. Panel (b) plots the sample cross correlation between these two series. The sample runs from 1960:M1 to 2016:M12.

**Time aggregation.** Finally, to link the monthly model-implied macro variables with the observed quarterly data, I use the following trend function:

$$
\Delta y_{qrt,o}^{i,t+1} = \sum_{j=1}^{5} \frac{3 - |j - 3|}{3} \Delta y_{i,t+2-j},
$$

(15)

such that the monthly model variables $\Delta y_{i,t+2-j}$ time-aggregate to the observed quarterly growth rates $\Delta y_{i,t+1}^{qrt,o}$, which are indicated by the $o$-subscript.\(^{12}\)

Overall, the combination of data available at different frequencies, where each series starts at a different period, temporal aggregation, and accounting for missing observations because of the real-time nature of the data set in conjunction with the regime changes, leads to a high-dimensional state-space model.

### 3.2 Estimation results

#### 3.2.1 Filtered common component and regime probabilities.
Panel (a) of Figure 2 shows the posterior mean estimate of the common component $z_t$, along with the NBER recession dates.

\(^{12}\)Older papers, Hansen and Sargent (1983) and Heaton (1995), and more recent ones, Bansal, Kiku, and Yaron (2016) and Schorfheide, Song, and Yaron (2018), account for time aggregation in their estimation in an asset-pricing setting.
To analyze the overall performance of the $z_t$ estimate, I take the Conference Board’s Economic Coincident Index as a reference time series of the in-sample current state of the business cycle. The figure depicts the year-on-year change in the Coincident Index. Notably, the real-time common component $z_t$ leads the Coincident Index by 3 to 5 months: the peak correlation is 0.7 at the 5 month lead (see panel (b) of Figure 2). Importantly, the common component $z_t$ can be interpreted as the investors’ real-time belief about the state of the business cycle.

Figure 2 shows that $z_t$ is clearly countercyclical: $z_t$ sharply drops in each ex post NBER recessionary period (see the light-gray shaded areas). Panel (a) of Figure 1 formalizes this point by plotting the posterior mean probability assigned to a period of broad economic contraction (i.e., $\tilde{\pi}_{t|t}$). Note that the posterior probabilities are in close agreement with the NBER index. At the start of each recession, the posterior mean recessionary probability experiences a marked increase. Moreover, based on the posterior distribution, I can compute the posterior mean unconditional regime probability of being in a recession; this probability equals $\pi_{ss} = 13\%$. I now proceed to describing the parameter estimates.

### 3.2.2 Parameter values for the stock return process.

Panel (a) of Table 1 reports quantiles for the posterior distribution of the model parameters and the statistics describing the overall model fit. These values are obtained in the in-sample estimation, that is, using all data available from 1930:M1 to 2016:M12. The quantiles for the priors are fairly agnostic. For the posterior distribution, the posterior median of $\rho$ is approximately 0.97, pointing toward a reasonable persistent predictable component in excess returns. The estimate of $\rho_{\mu,r}$ is -0.98, which is consistent with the implied values in Campbell (1991) and Campbell and Ammer (1993) and the assumed prior in Pástor and Stambaugh (2009). Notably, in the recessionary regime, expected returns considerably decreased.

---

13 The Coincident Economic Activity Index includes four indicators: nonfarm payroll employment, the unemployment rate, the average hours worked in manufacturing, and wages and salaries. The series can be found at https://fred.stlouisfed.org/series/USPHCI.

14 The NBER’s Business Cycle Dating Committee often determines this dummy variable with a substantial lag, which makes it contemporaneously unavailable to investors. For instance, the June 2009 business cycle was not announced by the NBER until September 20, 2010.

15 Most of the macroeconomic variables I use to compute the forecasting weights start in 1965:M1, so I completed the sample using the NBER recession index from 1930:M1 to 1964:M12. This early sample that uses the NBER index also serves to train the algorithm. That is, once new information arrives, it has to be classified as coming from either an expansionary or a recessionary regime. To this end, the optimal Bayesian procedure is using the information of previous NBER periods to classify the incoming information as belonging to either the expansion or the recession class.
Table 1. Posterior Parameter Estimates

This table reports the 5 and 95 percentiles of the prior distribution of the model parameter along with the 5, 50, and 95 percentiles of their posterior distribution. The priors for the parameters are fairly agnostic. With respect to the assumed prior distribution, N and U denote the normal and uniform distribution, respectively. The estimation sample runs from 1930:M1 to 2016:M12.

Next, I discuss how the two key parameters of the model, $\rho_{\varepsilon_{\mu,r}}$ and $\phi(S_t)$, account for the state-dependent autocorrelation in returns.

3.3 State-dependent autocorrelation in returns

3.3.1 Allowing for state-dependent autocorrelation. Given the assumed process for realized returns in equations (5), (6), and (7) and conditioning on a specific regime $S_t$, we can write the model-implied autocorrelation of returns as follows:

$$
\text{corr}_{S_t}(r_{t+1}^e, r_t^e) = \frac{\text{Cov}_{S_t}(r_{t+1}^e, r_t^e)}{\text{Var}_{S_t}(r_t^e)} = \rho^{h-1} \left(\rho \phi(S_t) + \rho_{\varepsilon_{\mu,r}} \sqrt{\phi(S_t)(1 - \phi(S_t))(1 - \rho^2)}\right). \quad (16)
$$

The autocorrelation of returns can be positive or negative, depending on the relative values of the model parameters. Panel (a) of Figure 3 illustrates this point by showing $\text{corr}_{S_t}(r_{t+1}^e, r_t^e)$ as a function of $\phi$ by fixing the rest of the parameters to its posterior median estimates (see Table 1). Note that the model-implied autocorrelation function is an increasing function of $\phi$, going from negative to positive values. This result can be understood by noting that two offsetting components...
of the assumed process of excess returns affect the autocorrelation function. The negative component arises because shocks to expected returns are accompanied by opposite shocks to current returns because $\rho_{\varepsilon, r} < 0$. The resultant negative serial correlation (expected from discount rate changes) tends to offset the positive serial correlation coming from persistent and variable expected returns and measured by the parameters $\rho$ and $\phi(S_t)$, respectively.

The central finding is that $\phi(S_t)$ is state dependent and higher in recessions. Specifically, the negative component dominates during periods of broad economic growth ($\phi(S_t = 0) \approx 1.1\%$ is relatively low) inducing a small negative return autocorrelation. In this state, a positive return shock induces a long string of mild negative returns that cumulatively bring prices back toward their original price. That is, we have mean-reversion. The circle in panel (a) of Figure 3 depicts the posterior median estimate for this case. In contrast, the positive component dominates in recessionary periods (note that $\phi(S_t = 1) \approx 9.6\%$ is relatively high) generating a strong positive return autocorrelation. Therefore, in recessions, a negative shock to returns sets off a string of negative returns, which add momentum to the original price decrease. The posterior median

---

Revisions in returns, $R_{N_t+1}$, are due to revisions in expectations about the current and the future dividend growth, $CFN_{t+1}$, or revisions in expected returns $DRN_{t+1}$, that is, $R_{N_t+1} = CFN_{t+1} - DRN_{t+1}$. From this equation, I see that a negative shock to expected returns will generate a positive return shock, unless a striking negative dividend growth shock gets in the way. Therefore, if the correlation between $CFN_{t+1}$ and $DRN_{t+1}$ is negative or sufficiently small, discount rate movements will generate a negative serial correlation in ex post returns. For returns to display momentum, I require a strong positive correlation between dividend growth and expected return shocks.
estimate for this state is depicted with a square in the figure. Finally, the knife-edge case happens when these two effects exactly offset each other and produce a zero autocorrelation of returns (depicted with a triangle). Hence, the autocorrelation of returns can be small (or zero) even under variable and persistent expected returns.

Next, I show how these two offsetting components affect the way the optimal filter uses the information in lagged returns to compute an estimate of the conditional expected return.

### 3.3.2 Exploiting the state-dependent autocorrelation.

To show how the model exploits the state-dependent autocorrelation of returns, I reexpress the aggregate estimate of expected returns in equation (11) as follows:

\[
E_t [r^e_{t+1}] = \text{Prob. expansion} \cdot \tilde{\pi}_{t|t} \cdot \text{Low } \phi(S_t) + \text{Prob. recession} \cdot \tilde{\pi}_{t|t} \cdot \text{High } \phi(S_t),
\]

where \(E_t [r^e_{t+1}|S_t = j]\) is the expected return computed from the predictive density associated with regime \(j\). Conditional on the economic regime \(S_t\), the model is linear. Hence, I can use Kalman-filtering steps to track the Gaussian distribution of the equity premium \(\mu_j\), where the \(j\) subscript indicates that I am conditioning on state \(j\).

The Kalman filter operates in two main steps. The first step consists of computing the expected value of \(\mu_j\), conditioned on the information set at time \(t - 1\), (i.e., \(r^e_{1:t-1}\)):

\[
\mu_{t|t-1}^j = E [r^e_{t+1}|\Theta, r^e_{1:t-1}, S_t = j] = E [\mu_t|\Theta, r^e_{1:t-1}, S_t = j] = \mu_0 + \rho (\mu_{t-1|t-1}^j - \mu_0).
\]

In the second step, I update the previous estimate \(\mu_{t|t-1}^j\) by incorporating the new information that arrived at time \(t\):

\[
\mu_{t|t}^j = E_t [r^e_{t+1}|\Theta, S_t = j] = E [\mu_t|\Theta, r^e_{1:t}, S_t = j] = \mu_{t|t-1}^j + \kappa (\Theta(S_t = j)) \left( r^e_t - \mu_{t|t-1}^j \right),
\]

Formally, \(E_t [r^e_{t+1}|S_t = j] = \int \int p(r^e_{t+1}|\Theta, r^e_{1:t}, S_t = j)p(\Theta|r^e_{1:t})dr^e_{t+1}d\Theta\).
where $\kappa(\Theta(S_t = j))$ represents the steady-state Kalman filter gain, which is a function of the parameters of the model and the regime in place.\footnote{I use the steady-state Kalman filter gain for the sake of simplicity. In general, the Kalman gain is also a function of time.} The weight given to the new information (i.e., $r_t^c - \mu_{jt|t-1}^j$) is precisely determined by the Kalman gain $\kappa(\Theta(S_t = j))$. Panel (b) of Figure 3 plots $\kappa$ as a function of $\phi$, and, as can be seen, $\kappa$ follows the same pattern as the autocorrelation function plotted in panel (a): it is negative (positive) for low (high) values of $\phi$, and it is equal to zero at the same $\phi$ value for which $\text{corr}_{S_t}(r_{t+h}^c, r_t^c)$ equals zero. Notably, under the posterior estimates, news about $r_t^c$ is negatively weighted in expansionary periods (low $\phi(S_t)$, see the circle in the figure) but positively weighted in recessionary times (high $\phi(S_t)$, depicted with a square).

### 3.3.3 Related to trend-following investment strategies

I can further express $\mu_{jt|t}$ as a weighted average of past returns (rather than past return news). To this end, I can substitute equation (18) into equation (19) and iterate backward:

$$
\mu_{jt|t} = \sum_{s=1}^{t} \delta_{s,t}(\Theta(S_t = j)) r_s \quad \text{with} \quad \sum_{s=1}^{t} \delta_{s,t}(\Theta(S_t = j)) = 1,
$$

(20)

where the Kalman-filtered derived weights on lag returns $\delta_{s,t}(\Theta_f,S_t)$ are a function of the Kalman gain $\kappa(\Theta(S_t = j))$. Panel (c) of Figure 3 plots these weights for three different cases. The first corresponds to the posterior median estimate of $\phi(S_t)$ in the expansionary regime. Under this case, returns exhibit time-series reversal (see panel (a)); the Kalman gain is negative (panel (b)); and lag returns are negatively weighted to predict future returns (panel (c)). Because the weights must sum to one, more distant lags are positively weighted (see the straight line). The second represents the weights when $\phi(S_t)$ is high, which happens during the recessionary regime. Here, returns exhibit time-series momentum; the Kalman gain is positive; and hence, all past returns are positively weighted to predict future returns. However, recent returns are more heavily weighted (see the dotted line). This case is akin to the time-series momentum strategy (e.g., Moskowitz, Ooi, and Pedersen, 2012; Hurst, Ooi, and Pedersen, 2017), which equally weights lag returns over a short period of time to predict future returns. The main difference is that the forecasting weights used here are optimal in the sense that they are derived from optimal filtering techniques. Finally, in the knife-edge case, returns are not autocorrelated; the Kalman gain is zero; and lag returns are...
equally weighted. Hence, the optimal estimate of the equity premium is equal to the unconditional
mean of $r^e$ (see the dashed line).

In sum, the benchmark forecasting strategy uses state-dependent optimal weights on lagged
return news to estimate the conditional expected return. Remarkably, in expansionary periods,
expected returns do not fluctuate much (i.e., $\phi(S_t)$ is low), returns exhibit reversal effects, and
the strategy uses time-series reversal weights on lagged return news to predict the next period’s
returns. Conversely, in recessions, expected returns exhibit considerable variations (i.e., $\phi(S_t)$ is
high), returns exhibit momentum, and the strategy uses momentum weights.

4 Economic relevance

I assess the economic relevance of my expected returns measure via a market timing strategy.
Specifically, I time the market by adjusting the portfolio weights according to the forecast of expected
returns. At the start of each month, the strategy increases risk-taking, when expected returns are
high, and decreases it, when expected returns are low. Thus, the return of the market timing
strategy is

$$r^e_{t+1} = c E_t[r^e_{t+1}] \cdot r^e_{t+1},$$

where the estimate $E_t[r^e_{t+1}]$ is formally defined in equation (11) and $r^e_{t+1}$ is the buy-and-hold excess
return. The constant $c$ controls the average risk exposure of the strategy and does not affect the
strategy’s Sharpe ratio. To facilitate the interpretation, I choose $c$, such that the managed return
has the same unconditional volatility as the buy-and-hold strategy.

All results that follow are obtained by estimating the model on recursively increasing samples
of real-time data. For my main results, I start the out-of-sample exercise in January 1980 and end
in December 2016.

4.1 Main results

Table 2 examines the economic significance of different specifications of the market timing trading
strategy. Column (1) reports results for the main specification. Panel (a) shows the risk-adjusted
Importance of the business cycle  

<table>
<thead>
<tr>
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<th>NBER</th>
<th>Steady</th>
<th>Exp.</th>
<th>Rec.</th>
<th>d/p ratio</th>
<th>Vol</th>
<th>Vol &amp; mkt</th>
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<td>Main foresight</td>
<td>S&lt;sub&gt;t&lt;/sub&gt; = 0</td>
<td>S&lt;sub&gt;t&lt;/sub&gt; = 1</td>
<td></td>
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Timing volatility

\[
\begin{align*}
 r_{t+1}^{e,\mu} &= \alpha + \beta r_{t+1}^{e} + \epsilon_{t+1} \\
\end{align*}
\]

Panel (a): Alphas controlling for the market factor

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<tbody>
<tr>
<td>(\alpha)</td>
<td>7.43</td>
<td>8.30</td>
<td>3.20</td>
<td>2.73</td>
<td>4.57</td>
<td>2.97</td>
<td>5.24</td>
</tr>
<tr>
<td>(2.50)</td>
<td>(2.86)</td>
<td>(1.86)</td>
<td>(1.44)</td>
<td>(1.74)</td>
<td>(0.94)</td>
<td>(2.59)</td>
<td>(3.95)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.49</td>
<td>0.40</td>
<td>0.76</td>
<td>0.73</td>
<td>0.39</td>
<td>0.12</td>
<td>0.70</td>
</tr>
<tr>
<td>(2.78)</td>
<td>(2.02)</td>
<td>(4.26)</td>
<td>(3.66)</td>
<td>(2.10)</td>
<td>(0.82)</td>
<td>(6.26)</td>
<td>(4.02)</td>
</tr>
</tbody>
</table>

Panel (b): Performance measures

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<tr>
<td>Sharpe increase (\times 100%)</td>
<td>58</td>
<td>62</td>
<td>23</td>
<td>12</td>
<td>6</td>
<td>-44</td>
</tr>
<tr>
<td>Performance fee (bps)</td>
<td>398</td>
<td>423</td>
<td>157</td>
<td>86</td>
<td>44</td>
<td>-303</td>
</tr>
</tbody>
</table>

Table 2. Performance evaluation of timing the market

This table presents results of the economic significance of the market timing strategy. I consider eight different strategies, which I describe in the main text. All models are estimated out of sample on recursively increasing samples of real-time data. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. To facilitate an interpretation, I annualize all returns, expressed as a percentage per year, by multiplying monthly returns by 12. The forecast evaluation period runs from 1980:M1 to 2016:M12.

The main forecast delivers a large and significant alpha relative to the market of about 7.4% (annualized). The beta of only 0.49 suggests that the strategy essentially hedges market risk. Panel (b) of Table 2 reports various profitability measures. The annualized appraisal ratio, \(\alpha/\sigma_\epsilon\), of the strategy is 0.55. This number translates into an annualized Sharpe ratio of 0.69 \(= \sqrt{SR_{Mkt}^2 + (\frac{\alpha}{\sigma_\epsilon})^2}\), which is economically large considering that the market’s Sharpe ratio is around 0.44 for the 1980-2016 sample period. These numbers translate into an overall 58% increase in the buy-and-hold Sharpe ratio.

An alternative performance measure that captures the trade-off between risk and returns is via a utility-based approach. Following Fleming, Kirby, and Ostdiek (2001), I equate the realized quadratic utility obtained under the dynamic portfolio strategy with the one obtained under the static buy-and-hold strategy,

\[
\begin{align*}
 T^{-1} \sum_{t=0}^{T-1} \left[ (r_{t+1}^{e,\mu} - \Delta) - \frac{\gamma}{2(1 + \gamma)} (r_{t+1}^{e,\mu} - \Delta)^2 \right] &= T^{-1} \sum_{t=0}^{T-1} \left[ (r_{t+1}^{e}) - \frac{\gamma}{2(1 + \gamma)} (r_{t+1}^{e})^2 \right] \\
\end{align*}
\]

Specifically, I run time-series regressions of the managed return on the aggregate excess market return (i.e, \(r_{t+1}^{e,\mu} = \alpha + \beta r_{t+1}^{e} + \epsilon_{t+1}\)).
and solve for $\Delta$ for a given level of risk aversion $\gamma$. The parameter $\Delta$ can be interpreted as the monthly maximum performance fee that an investor would be willing to pay to have access to the dynamic strategy. The performance fee for the main strategy is substantial. For instance, an investor with a quadratic utility and a risk aversion coefficient of 3 is willing to pay a monthly annualized fee of around 398 basis points (bps) to have access to the $E_t[r_{t+1}]$ estimate.

The left panel of Figure 4 depicts the cumulative excess return of the main strategy from 1980 to 2016. As the figure shows, the main strategy (dark-blue line) generates a relatively steady stream of positive returns that consistently outperform the fully invested buy-and-hold strategy (light-blue line). For instance, given a $1 investment in January 1980, the managed strategy would generate a value of around $30, which exceeds the $10 value generated by the fixed allocation benchmark. Finally, the right panel of Figure 4 plots the drawdown to better explain when the strategy incurred losses. The main strategy’s time under water is usually concentrated in recessionary periods but reverts shortly after. Likewise, the buy-and-hold approach incurs large market losses during recessions; however, it requires a long period to recoup these losses. This result illustrates that my strategy is able to accurately identify business cycle turning points and dynamically adjust the

---

More generally, the quadratic utility function can be viewed as a second-order approximation to the investor’s utility function. Throughout the paper, I assume a risk aversion coefficient of 3 to compute the performance fee. However, I verified that the outperformance is robust to different perturbations to $\gamma$. 

---

Figure 4. Cumulative gains and drawdowns.
The left panel plots cumulative monthly gains for the main specification (dark-blue line) and the static buy-and-hold strategy (light-blue line). I also show results for the strategy that assumes perfect NBER foresight; that is, I know in real time the NBER recessionary index (light-orange line) and for the market timing strategy that uses the dividend-price ratio as a predictor (dark-orange line). The right panel depicts the drawdown of each strategy. The sample runs from 1980:M1 to 2016:M12.
portfolio weights to avoid being caught flat-footed to unfavorable shifts in the economic outlook.

In the rest of this section, I show that the superior market timing abilities of the main specification come from being able to identify the state of the business cycle in a timely fashion and by exploiting the state-dependent autocorrelation in returns.

4.2 How important is the state of the business cycle?

Columns (2)–(5) in Table 2 report the results. I start by showing results for the hypothetical scenario that assumes perfect NBER recessionary foresight. This exercise can be thought of as reflecting a situation in which investors could perfectly foresee recessions with the ex post NBER accuracy. Column (2) of Table 2 shows that the ex post NBER economic state provides an increase in alphas relative to the main strategy. For instance, it produces an alpha of 8.3% and a 62% increase in the buy-and-hold Sharpe ratio. Moreover, the added value of the NBER perfect foresight strategy is $40, which favorably compares with the $30 achievable in real time (see the light-orange line in Figure 4).

This result is reassuring, because it suggests that having better measures of business-cycle turning points increase the strategy’s overall performance. In essence, the differences in out-of-sample gains measure the economic relevance of the added informational content.

The models under columns (3), (4), and (5) do not condition on the state of the business cycle. In column (3), I do not allow for time-varying recessionary probabilities in the measure of expected returns. I do this by fixing \( \pi_{t|t} \) to its unconditional value of \( \pi_{ss} \approx 13\% \) for each period \( t \) in equation (11). Alternatively, the forecasts under column (4) only consider the predictive density associated with the low variability of expected returns (i.e., low \( \phi(S_t) \) in both states; mild reversals in returns; and negative forecasting weights). Finally, the forecasts under column (5) consider the predictive density associated with the high \( \phi \) (i.e., high \( \phi(S_t) \) in both states; strong momentum in returns; positive forecasting weights).

Notably, these three alternative models also can generate profits. For instance, they produce an overall 23%, 12%, and 6% increase in the buy-and-hold Sharpe ratio, respectively. However, these gains are an order of magnitude smaller than the ones obtained under the main specification.

---

21 To this end, I use the ex post NBER recessionary index for the state of the business cycle rather than the real-time recessionary probability estimates \( \pi_{t|t} \) and estimate the model again.

22 To compute a measure of expected returns for these alternative models, I took the out-of-sample parameter estimates associated with each regime and filtered (using those values) an estimate of expected returns.
Table 3. Out-of-sample $R^2_{OOS}$ across NBER economic states
This table presents the out-of-sample $R^2_{OOS}$ across NBER economic states. All models are estimated out-of-sample on recursively increasing samples of data. The forecast evaluation period runs from 1980:M1 to 2016:M12.

Where are these gains coming from? To help us understand where these gains are coming from, Table 3 reports the out-of-sample $R^2$ statistic, $R^2_{OOS}$, computed over the whole sample period and the $R^2_{OOS}$ obtained using only recessionary and expansionary periods, as measured by the NBER recession indicator. The main specification delivers one of the strongest predictors of the equity risk premium identified to date, with the monthly $R^2_{OOS}$ reaching 2.0% ($p$-values below 0.01). Consistent with the literature, most of my predictability comes from recessionary periods ($R^2_{OOS,Rec.} \approx 5.0\%)$. In contrast to this literature, the striking degree of the predictability of my strategy predominantly stems from being able to also predict market returns during expansions with an $R^2_{OOS}$ of around 1.3%. Essentially, the main strategy allows for state-dependent time variation in expected returns (measured by $\phi(S_t)$); thereby stabilizes the forecast; and prevents overfitting in precisely those periods when return predictability is low (e.g., Cujean and Hasler 2017).

Alternative models predict well in some states but perform poorly in other states. For instance, the model in column (5) that assumes strong momentum in returns is able to predict very well in recessions with monthly $R^2_{OOS}$ values of around 6%, but performs poorly in expansions with negative statistics of -1% (a negative $R^2_{OOS}$ means that the simple recursive mean forecast produces

23 The out-of-sample $R$-squared statistic is computed as follows: $R^2_{OOS} = \left( 1 - \frac{\sum_{i=m+1}^T (r^e_t - \hat{r}^e_{t,c})^2}{\sum_{i=m+1}^T (r^e_t - \bar{r}_t)^2} \right) \times 100\%$, where $\bar{r}_t$ is the historical average return estimated through period $t-1$. If $R^2_{OOS}$ is positive, I say that the forecast $\hat{r}^e_{t,c}$ outperforms the historic average forecast based on a sum of squared forecast errors metric.


25 These are large statistical values for out-of-sample predictions, comparable in magnitude to the unconditional estimates reported in previous literature, such as the monthly $R^2_{OOS}$ of around 0.93% in Kelly and Pruitt (2013), who use a single factor extracted via partial least squares from 100 book-to-market ratios.
lower forecasting errors). Conversely, the models that assume mild reversals in returns are able to predict well in expansions ($R^2_{OOS,Exp.} \approx 1.7\%$), by sacrificing the predictability in recessions ($R^2_{OOS,Rec.} \approx -0.98\%$).

Substantial profits, together with the large $R^2_{OOS}$ statistics across economic states, suggest that a strong feature of the U.S. stock market is that it exhibits mild mean-reversion in periods of broad economic growth, but times-series momentum during contractions. Furthermore, a method that exploits this pattern to compute a measure of expected returns—using optimal filtering techniques—can generate significant market timing abilities.

### 4.3 Comparison with common alternative predictors

The performance of the main specification may be surprising given the prevailing tone of an extensive market timing literature showing that return predictability is highly unstable and concentrated in relatively short-lived periods (for a recent study, see Farmer et al. 2018). To illustrate this point, I take the log dividend-price ratio ($d/p$) as representative of the vast predictors documented in the literature and use it to time the market. The strategy’s performance is disappointing when using the $d/p$ predictor, but this result is expected, given the results of Goyal and Welch (2008). Even before costs, the timing strategy cannot beat the fully invested buy-and-hold approach with an added value of only $2.5$ (see the dark-orange line in panel (a) of Figure 4). Likewise, it decreases the Sharpe ratios, generates negative performance fees (see Table 2), and produces negative $R^2_{OOS}$ (see Table 3).

I conceptualize the main specification as providing a good measure of expected returns over monthly horizons, whereas traditional predictors, such as the dividend-price ratio, measure expected returns over longer horizons.

### 4.4 Macro or financial variables?

The filtered common component, $z_t$, is based on a broad set of information about economic conditions, including forward-looking financial variables. As shown in Section 3.2.1, $z_t$ leads standard coincident indicators by 3–5 months. Hence, $z_t$ could provide early signals of turning points in the business cycle that are not fully reflected in market prices, rendering a market timing strategy profitable. Next, I analyze the sensitivity of the strategy with respect to the macro and financial...
Table 4. Performance evaluation after restricting the information set

This table presents results for the market timing strategy that restricts the information set used to identify the real-time recessionary probabilities \( \pi_t \). The big 4 include four coincident indicators: GDP, Industrial Production Index, Nonfarm Payroll Employment, and Real Personal Income Excluding Current Transfer receipts. In column (2) I consider the big 4 plus other macro indicators such as Initial Jobless Claims, Weekly Hours Worked, CPI, and Consumer Sentiment. For a complete list of the variables used see Section 3.1. All models are estimated out of sample. Standard errors are adjusted for heteroskedasticity and autocorrelation. \( t \)-statistics are reported in parentheses. To facilitate an interpretation, I annualize all returns, expressed as a percentage per year, by multiplying monthly returns by 12. The forecast evaluation period runs from 1980:M1 to 2016:M12.

Variables used to compute \( \pi_{t|t} \). Table 4 shows these results.

Column (2) presents results for the strategy that only considers the information contained in four macroeconomic indicators to compute the recessionary probability \( \pi_t \). Specifically, I consider the following coincident indicators: GDP, Industrial Production Index, Nonfarm Payroll Employment, and Real Personal Income Excluding Current Transfer Receipts. These indicators tend to move with the current state of economic activity and are widely follow by practitioners. Hence the name the big 4. The strategy’s performance deteriorates, but not by much, surprisingly. The restricted strategy produces an alpha of 5%, a performance fee of 354bps, a 50% increase in the buy-and-hold Sharpe ratio, and a \( R^2_{OOS} \) of 1.28%. As shown in Column (4), adding the rest of the macro indicators to the big four (e.g., Initial Jobless Claims, Weekly Hours Worked, CPI, Consumer Sentiment) increases the strategy’s performance, with values relatively close to the ones achieved under the main specification.

Overall, these results suggest that macroeconomic variables play a relatively important eco-
nomic role in identifying business-cycle turning points. Removing the information contained in the financial variables has a minor effect on the strategy’s overall performance.

### 4.5 Timing volatility

Next, I show that the strategy’s performance can be improved by timing volatility. Specifically, I modify the portfolio weights in (21) by scaling the expected return by the inverse of the conditional variance. The managed portfolio return is then

\[
r_{t+1}^{e,\mu-\sigma} = c \frac{E_t [r_{t+1}^e]}{\sigma_t^2} \cdot r_{t+1}^e,
\]

where again I set \(c\) such that \(r_{t+1}^{e,\mu-\sigma}\) has the same unconditional volatility as \(r_{t+1}^e\). The main motivation for the use of these weights comes from the optimal dynamic strategy of a mean-variance investor who levers her position up or down on the risky asset over time, to maximize the unconditional Sharpe ratio of the portfolio (see Daniel and Moskowitz 2016). To compute the weights, I use the main specification for \(E_t [r_{t+1}^e]\). For \(\sigma_t^2\), I follow Moreira and Muir (2017) and use the previous month’s realized variance, which is computed from the daily data.\(^{26}\)

Column (8) of Table 2 reports the results. The strategy generates an alpha of 7.6%, a Sharpe ratio increase of 65%, and a substantial performance fee of 460 bps. These numbers favorably compare to the strategies that time either expected returns (using the main forecast) or volatility (using \(c \sigma_t^2\)) as shown in columns (1) and (7), respectively.

### 4.6 Battery of robustness tests

To assess the robustness of the main result, I perform an extensive battery of tests. I will briefly describe them here, but the appendix provides extensive details. I document that the strategy generates similar alphas across different subsample periods for a wide range of pricing factors. I also show that it produces fairly large benefits under tight leverage and short-selling constraints, even if I preclude the strategy to use them. Moreover, transaction costs would need to be incredibly high to make the strategy unprofitable. Furthermore, the results remain the same when I impose

\[^{26}\text{Specifically, I compute } \sigma_t^2 \text{ as follows: } \sigma_t^2 = \sum_{d=1}^{T_{d,t}} \left( r_{d,t}^e - \frac{\sum_{d=1}^{T_{d,t}} r_{d,t}^e}{T_{d,t}} \right)^2, \text{ where } t \text{ denotes the month and } T_{d,t} \text{ the total number of days in month } t.\]
stale information of 1 or 2 months in the forecasted regimes. Nevertheless, profitability during the recessions significantly deteriorates beyond the second-month gap. This last result highlights the importance of identifying the regimes in a timely fashion. I further assess the economic relevance of the two key assumed model parameters (i.e., $\rho_{\xi_{t},r}$ and $\phi(S_t)$) by turning off each of them at a time and reestimating the model. I find that for both cases the profitability of the market timing strategy disappears.

So far, I have used alphas, appraisal ratios, Sharpe ratios, utility-based performance fees, and out-of-sample $R$-squareds to gauge my strategy’s ability to time the market. I show that the return-timing strategy’s performance is also robust to the regression-based timing measures of Treynor and Mazuy (1966) and Henriksson and Merton (1981). The strategy is also profitable if I use the manipulation-proof measures of performance of Goetzmann, Ingersoll, Spiegel, and Welch (2007) or as measured in the Morningstar Risk Adjusted Rating. I document similar profitability for characteristic-sorted portfolios, such as value, size, and momentum. Using lag information about the NBER index, I extend the analysis to the 1960–2016 sample period and show similar outperformance for this earlier sample. Similarly, using equity returns from ten countries, I find by and large consistent results. Importantly, in the appendix, I also show that movements in expected returns are not spuriously driven by movements in cash flows.

For completeness, I also report a number of return spreads for which my forecasting strategy does not predict well. In particular, I consider the size, value, momentum, profitability, and investment factors. For most of the pricing factors the alphas are economically small and statistically insignificant. The only exception is on the momentum factor, for which the market-timed momentum factor has an annualized alpha of around 6% (46% Sharpe increase), and a beta of 0.60. If the forecasting strategy does a good job of capturing movements in the aggregate market, it is, perhaps, unsurprising that the strategy does not work well for return spreads that hedge market risk.

Finally, in a companion paper (Gomez Cram 2017), I compare the predictability ability of my forecasting strategy with the extensive stock return predictability literature. Overall, I conclude that exploiting the state-dependent predictability and autocorrelation of returns delivers one of the

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27The Morningstar methodology can be found at [https://corporate.morningstar.com/US/documents/MethodologyDocuments/FactSheets/MorningstarRatingForFundsFactSheet.pdf](https://corporate.morningstar.com/US/documents/MethodologyDocuments/FactSheets/MorningstarRatingForFundsFactSheet.pdf)

28For this result, I use the 1930–1959 sample period as a training sample and start the out-of-sample forecast in January of 1960.
strongest predictors of the equity risk premium identified to date. In contrast with the previous literature, my results suggest that stock return predictability is neither a short-lived nor a recessionary phenomenon, but a strong feature of the U.S. stock market.

Taken altogether, the evidence for the U.S. aggregate stock market, including the extended sample period, together with the results for every characteristic-sorted portfolio or for every international stock market indices that I examine, supports the $E_t[r_{t+1}]$ estimate as a good measure of expected returns over monthly horizons.

5 Risk exposure across the business cycle

In this section I analyze the behavior of expected returns across the business cycle. I start by running contemporaneous time-series regressions of the strategy’s risk exposure (i.e., $\omega_t \propto E_t[r_{t+1}]$) onto five variables related to future stock market returns as previously documented in the literature.

The first variable that I consider is investors’ expectations of future stock market returns, obtained from the American Association of Individual Investors (AAII) survey. Each week since 1987, the AAII asks its members a simple question: "Do you feel the direction of the stock market over the next six months will be up (bullish), no change (neutral) or down (bearish)?" My measure of expectations is the monthly average of the percentage of "bullish" investors minus the percentage of "bearish" investors (i.e., $\%_{\text{Bull}} - \%_{\text{Bear}}$). Greenwood and Shleifer (2014) also rely on the AAII survey data. My second variable is the investor sentiment measure of Baker and Wurgler (2006). My third variable is the monthly investor inflows into equity-oriented mutual funds from the Investment Company Institute. In each period, I scaled the inflows by the aggregate capitalization of the U.S. stock market. The fourth variable is the year-on-year change in the Coincident Index, which measures the current state of the business cycle. The fifth and last variable is the dividend-price ratio, which is a typical measures of risk premia in the literature. I standardized these variables to ease the interpretation of the regression coefficients.

Table 5 presents the results. As shown in Columns (1), (2), and (3), during NBER expansions, when the survey measure of return expectations, sentiment, or flows into the equity markets increase, the strategy takes a contrarian position and reduces risk-taking. In good times, these results are consistent with the findings of Greenwood and Shleifer (2014), who show that six investor surveys...
I run time-series regressions of the strategy’s risk exposure on various contemporaneous variables $\omega_t = (\omega_0 + \omega_{rec} \times I_{rec,t}) + (\beta_0 + \beta_{rec} \times I_{rec,t}) r_t + \epsilon_t$. The risk exposure is proportional to the estimate of expected returns $\omega_t \propto E_t[r_{t+1}]$ described in Section 2 and $I_{rec,t}$ denotes the NBER recessionary index. The data are monthly on the sample period 1987 to 2016 given that the %Bull - %Bear series from the American Association of Individual Investors survey starts in 1987. The sentiment measure of Baker and Wurgler (2006) ends in 2015:M9. Standard errors are adjusted for heteroskedasticity and autocorrelation. $t$-statistics are reported in parentheses.

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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<td>0.81</td>
<td>0.77</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(9.84)</td>
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<td>(8.41)</td>
<td>(10.32)</td>
<td>(8.80)</td>
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<td>-0.14</td>
<td>-0.36</td>
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</tr>
<tr>
<td></td>
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<td>(-1.38)</td>
<td>(0.15)</td>
<td>(-0.47)</td>
<td>(-1.66)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>$R^2$-adjusted</td>
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<td>0.20</td>
<td>0.22</td>
<td>0.14</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Table 5. Regression of risk exposure on contemporaneous variables.

I run time-series regressions of the strategy’s risk exposure on various contemporaneous variables $\omega_t = (\omega_0 + \omega_{rec} \times I_{rec,t}) + (\beta_0 + \beta_{rec} \times I_{rec,t}) r_t + \epsilon_t$. The risk exposure is proportional to the estimate of expected returns $\omega_t \propto E_t[r_{t+1}]$ described in Section 2 and $I_{rec,t}$ denotes the NBER recessionary index. The data are monthly on the sample period 1987 to 2016 given that the %Bull - %Bear series from the American Association of Individual Investors survey starts in 1987. The sentiment measure of Baker and Wurgler (2006) ends in 2015:M9. Standard errors are adjusted for heteroskedasticity and autocorrelation. $t$-statistics are reported in parentheses.

broadly consistent with studies in the behavioral literature suggesting that many investors hold extrapolative expectations about returns. After a period of good stock market performance, one that generates high sentiment, many investors will become more bullish. Therefore, extrapolators (including the AAII survey) are negatively correlated with traditional predictors of returns, such as the dividend-price ratio.

Survey evidence suggests that investors form beliefs about future returns by extrapolating past market performance. See, for instance, Vissing-Jorgensen (2003), Amromin and Sharpe (2009), Bacchetta, Mertens, and Van Wijncoop (2009), and Greenwood and Shleifer (2014). For theoretical models with investors that extrapolate past returns, see Cutler, Poterba, and Summers (1990), De Long, Shleifer, Summers, and Waldmann (1990), Hong and Stein (1999), Barberis and Shleifer (2003), and Barberis et al. (2015).
will rebalance their portfolios into equity, and, doing so, will bid up current stock prices even further. At this point, the stock market is arguably overvalued, and, hence, subsequent price changes are low, on average. In essence, the strategy exploits this mispricing by mildly decreasing the portfolio weights on stocks. For instance, see from panel (c) of Figure 3 that during expansions the strategy negatively weights lag returns to predict future returns. This is by and large consistent with the equilibrium model of Barberis et al. (2015) in which rational traders counteract the overvaluation caused by extrapolators. Therefore, my expected return estimates can be conceptualized as reflecting the return expectations of rational traders in their model. In contrast, extrapolation of returns does not explain my strategy’s profits during recessions. Note that during these bad economic times the strategy follows the survey and sentiment measures as well as the flows into equity markets. Although for the former two series, the point estimates are close to zero.

During expansions the strategy’s profits also can be explained in terms of risk. Columns (4) and (5) show that during expansions, when economic conditions deteriorate (but not enough to generate a recession) or when prices decrease more than dividends (representing an increase in risk premia), the strategy increases risk-taking. These results imply that during expansions, the strategy has a strong countercyclical component. Hence, profits also can be viewed as compensation for risk bearing.31

While during expansions I cannot determine whether the strategy’s profits are due to market inefficiency or as a result of rational variation in expected returns, my profits during recessions conflict with standard risk-based explanations. Specifically, during recessionary periods, an increase in the dividend-price ratio or a decrease in macro growth would suggest high expected returns and thus a time to buy stocks. Yet my strategy rebalanced the portfolio weights out of equity and into Treasuries. Next, I show that the strategy earns the bulk of its profits by taking less risk at the onset of a recession and by aggressively getting back in shortly afterward.

31 For example, in a standard representative agent asset pricing model, risk premia vary over the business cycle and are relatively high in bad times, when the economy is slowing down. For habit models, see Campbell and Cochrane (1999). For long-run risk models, see Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). For models with rare disasters, see Rietz (1988), Barro (2006), Gabaix (2012), and Wachter (2013).
Table 6. Market timing regression results.
This table presents risk exposure and alphas of the strategy during recessions. I regress the managed return on a constant, the NBER recession dummy $I_{rec,t}$, and an indicator for bull markets $I_{b,t}$ that equals 1 if the cumulative past 6-month return on the market (including time $t$) is positive:

\[ r_{t}^{e,\mu} = (\alpha_0 + \alpha_{rec} I_{rec,t}) + \beta_0 r_t + \beta_{rec} I_{rec,t} + \beta_{rec,b} I_{b,t} + \epsilon_t. \]  

Standard errors are adjusted for heteroskedasticity and autocorrelation. $t$-statistics are reported in parentheses. To facilitate an interpretation, I annualize all returns, expressed as a percentage per year, by multiplying monthly returns by 12. The sample runs from 1980:1 to 2016:12.

5.1 Risk exposure during recessions
Successfully timing the market requires two correct decisions. The first depends on exiting the market. To see when the strategy cashes out, I start by fitting a conditional CAPM with the NBER recessionary indicator, $I_{rec,t}$, as an instrument:

\[ r_{t}^{e,\mu} = (\alpha_0 + \alpha_{rec} I_{rec,t}) + \beta_0 r_t + \beta_{rec} I_{rec,t} + \epsilon_t. \]  

This specification captures both risk-adjusted returns and market beta differences across economic states. Column (3) in Table 6 shows the results. A striking change is evident in the beta in recessionary periods. It is -0.81 lower ($t$-statistic = -2.14), suggesting that the strategy reduces risk-taking during recessions by completely cashing out. Indeed, the beta of the strategy, conditional on a recession, equals -0.14 ($=\beta_0 + \beta_{rec}$). Interestingly, this change in risk exposure explains almost half of the alpha differences in recessions, which shrinks from 17.6% (see column (2)) to 9.6%. Although not statistically significant, it is still economically large.

The second decision is when to reenter. To gauge the strategy’s ability to adjust when the market rebounds during economic downturns, I introduce an additional term to the previous
specification:

\[ r_{t}^{\text{cum}} = (\alpha_0 + \alpha_{\text{rec}} I_{\text{rec},t}) + (\beta_0 + (\beta_{\text{rec}} + \beta_{\text{rec,b}} I_{b,t}) I_{\text{rec},t}) r_{t}^{e} + \epsilon_t, \]  

(25)

where \( I_{b,t} \) is an indicator that equals 1 if the cumulative return in the past 6 months (including time \( t \)) is positive and 0 otherwise. A \( \beta_{\text{rec,b}} \) of 1.55 (t-statistic = 3.95) in column (3) shows that the strategy is able to get back in once the market rebounds. In recessionary periods, the risk exposure of the strategy is -0.24 \((-\beta_0 + \beta_{\text{rec}}\) during down-markets, but 1.31 \((-\beta_0 + \beta_{\text{rec}} + \beta_{\text{rec,b}}\) during up-markets. Notably, this change in risk exposure during recessions explains the alpha difference with a point estimate of \( \alpha_{\text{rec}} \) equal to -0.03.

The key insight is that down- and up-markets do not happen randomly during recessions. Panel (a) of Figure 5 makes this point by plotting the portfolio weights during recessions. The vertical bars correspond to 1 standard error above and below the point estimates. Expected returns are substantially negative for several months following the onset of a recession, before rising to levels that are persistently higher than average. Notably, these portfolio dynamics align with the realized excess returns of the simple trading strategy described in the introduction and shown in panel (b) of Figure 1.

Reducing risk-taking during bad times is consistent with Moreira and Muir (2017), who show that volatility-managed portfolios also cash out during recessions. However, one main difference between timing the market and timing volatility is on when to get back in. The volatility strategy will slowly return to the market only when the volatility starts to decrease, not necessarily when the market rebounds (see panel (a) of Figure 5). Therefore, volatility-timing strategies may stay out of the market for too long and potentially miss the subsequently higher-than-average excess returns. Indeed, zeroing in on these recessionary periods, panel (b) and (c) of Figure 5 show the cumulative monthly returns to these different strategies. I show only results for the subsamples surrounding the July 1981 - November 1982 and December 2007 - June 2009 recessions. Over both of these subsamples, the market-timed strategy outperforms the other strategies.

---

32 The results remain largely unchanged for different look-back time frames around the 6-month benchmark period.
33 In fact, the point estimates of the regression coefficient \( \beta_{\text{rec,b}} \) in equation (25) is equal to 0.10 (t-statistic = 1.17) for the volatility timing strategy and equal to 0.40 (t-statistic = 3.99) for the market and volatility timing strategy.
34 Similar results can be found in the other recessions contained in my sample period, that is, the recessions of July 1990–March 1991 and March 2001–November 2001. I present results for these two recessions, because they are the first and last recessionary periods in my out-of-sample window.
Figure 5. Portfolio weights and cumulative gains during recessions.
The left panel depicts the portfolio weights during the expected economic contractions. The diamonds correspond to the point estimates, and the vertical bars represent 1 standard deviation error bars and account for heteroscedasticity and autocorrelation in the residuals. The estimation sample runs from 1980:M1 to 2016:M12. I consider the portfolio weights of four investment strategies: (1) market timing, (2) market and volatility timing, (3) volatility timing, and (4) a buy-and-hold strategy. The two panels at the right show the monthly cumulative returns of these four strategies over the periods January 1981–December 1983 (panel (b)) and January 2007–December 2010 (panel (c)).

Expected returns are higher in recessions. How can I reconcile my findings of reducing risk-taking at the onset of bad expected times with the well-known empirical evidence that shows that expected returns are higher in recessions? As Section 5 shows, at the start of recessions, expected returns are negative. However, they are subsequently followed by higher-than-average expected returns (see Figure 1). Given that the former period is shorter than the latter period, the overall cumulative return is positive. For instance, a back-of-the-envelope calculation (based on the numbers from Figure 1) shows that a simple strategy that invests in the market at the start of an expected recession and stays put for 2 years delivers an annualized performance of 8.0%, which compares favorably with the unconditional equity premium of 6.8% in my sample. Thus, my results are not in conflict with the evidence that expected returns are higher in recessions. However, as I have been arguing, the first months into a recession are particularly bad times to invest in stocks and an active strategy would like to cash out during these times and reenter in shortly afterward.

Overall, these results suggest a mismatch between business-cycle risk and compensation for
bearing macroeconomic risk in the time series.

5.2 Implications for general equilibrium models

The evidence presented so far raises a puzzle. In Section 4 I have argued –based on the economically significant profits of the market timing strategy– that the econometric model provides a good measure of expected returns. Whereas in Section 5.1 I have shown that expected returns are, if anything, on average, negative or low at the onset of expected recessions. In this section, I will discuss the implications of this evidence for both rational and behavioral models.

5.2.1 Risk-based theories. From a theoretical point of view, it is difficult to rationalize negative equity premia within any equilibrium model. In rational asset pricing models, risk premia are given by the negative covariation between news to returns and news to the stochastic discount factor (SDF). For instance, under standard lognormality assumptions, the conditional market risk premium can be written as follows:

\[ E_t(r_{t+1}^e - r_{f,t}) + \frac{1}{2} \text{var}_t(r_{t+1}^e) = -\text{cov}_t(m_{t+1}, r_{t+1}^e), \]  

(26)

where \( m_{t+1} \) is the log SDF, which is a function of state variables. For example, the key state variable in the habit formation models (e.g., Campbell and Cochrane 1999) is the surplus consumption ratio of the representative agent, which measures how far is her consumption from her habit (which, in turn, is a function of current and past consumption). In essence, as consumption declines relative to habit, risk aversion rises, resulting in an increase in the conditional equity premium. Likewise, in standard dynamic models with long-run risk (e.g., Bansal et al. 2012, Bansal and Yaron 2004), the log SDF is a function of expected consumption growth and its time-varying volatility. In these models, the equity premium is driven by fluctuations in the conditional volatility of expected consumption growth, implying that expected returns are high when real uncertainty is also high. Similarly, in models with rare disasters (e.g., Gabaix 2012, Wachter 2013) the conditional equity premium is a function of a time-varying probability of a disaster, which is characterized by a large drop in consumption. Thus, expected returns increase when the expected probability of disaster increases. The bottom line is that in all these asset pricing models, the compensation for risk is
precisely high—not negative or low—in periods of broad expected macroeconomic contractions and large uncertainty.

Intuitively, from the null of leading risk-based asset pricing models, selling at the start of expected recessions reduces the overall risk, but at the same time sacrifices high expected returns. Hence, the alpha of my strategy should be, in any case, zero[35] yet I show that it is economically large. Moreover, given that the model-implied equity premium at the start of an expected recession has the wrong sign, relative to the sign found in the data (as opposed to a quantitative mismatch, for instance), these models will not be able to overcome the inconsistency by changing some parameter values in the original calibrations. Therefore, rather than replicating my empirical analysis on model-simulated data, I directly conclude that these risk-based models cannot fully explain the facts discussed thus far. However, I also recognize that these models were designed to capture time variation in risk premia at lower frequencies than the one studied here.

5.2.2 Behavioral theories. Negative expected returns in precisely bad times are difficult to explain in terms of risk. My findings during these recessionary periods, however, appear to be more consistent with the extensive body of empirical literature suggesting that security prices underreact to news. Most of the literature focuses on the cross-section of stock returns and reports that prices drift over horizons between 1 to 12 months after major news announcements or corporate events. My findings complement this anomalous evidence by showing that aggregate stock prices seem to be late in fully reflecting all information that signals the onset of recessions. As a consequence, stock prices subsequent to the start of a recession exhibit a predictable drift, which generates the strong positive return autocorrelation observed during bad times. In essence, my strategy earns superior returns by exploiting the underreaction and even reducing risk-taking at the same time.

Nevertheless, my strategy’s profits also present a challenge to behavioral theory, because current models do not simultaneously explain all facts. For instance, the findings about recessions are broadly consistent with the models of Barberis et al. (1998) and Daniel et al. (1998), who suggest that preferences specifications based on conservatism or self-attribution can play an important role

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[35] To see this note that all conditional alphas are zero. In essence, timing the market is a managed version of the market return \( E_t[\omega_t r_{t+1}] = \beta_t \mu_t \), where \( \beta_t = \omega_t \) and \( \mu_t = E_t[\hat{r}_{t+1}] \). From here, we can write the unconditional alpha as \( \alpha = (E(\beta_t) - \beta) \mu + \text{cov}(\beta_t, \mu_t) \), with \( \mu = E[\hat{r}_{t+1}] \). Therefore, if a strategy prescribes to reduce risk (i.e., a decrease in \( \beta_t \)), when expected returns are high (i.e., \( \mu_t \) is high), then the covariance term will be negative, which reduces the overall alpha of the strategy.
5.3 Who is late to recessions?

Previous results suggest that stock prices do not fully incorporate the information that signals the onset of a recession, but, in equilibrium, prices reflect investors’ demands, thereby raising the question of “who is late to recessions?” What type of investors does not trade aggressively enough in response to macro information? Although a definitive answer to this question requires data on stock market positions, previous research suggests that investors act in line with their expectations of future stock market returns as measured by surveys (e.g., Greenwood and Shleifer (2014)).

Panel (a) of Figure 6 plots the AAII survey of investors’ expectations during recessions. At

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36 Barberis et al. (1998) defines conservatism "as the slow updating of models in the face of new evidence." On the other hand, Daniel et al. (1998) define self-attribution as the phenomenon documented in psychology in which "individuals too strongly attribute events that confirm the validity of their actions to high ability, and events that disconfirm the action to external noise or sabotage."
the onset of recessions, the overall sentiment (i.e., %Bull - %Bear) about the future performance of the stock market is at its unconditional average (denoted by the vertical line). Investors become pessimistic about the stock market and reflect this opinion about their expectations for macroeconomic contraction at around 6 months. Subsequently, the overall sentiment returns to its unconditional value. Panels (b) and (c) of Figure 6 further zeros in on the type of investors who are more likely to underreact. Panel (b) identifies that investors who are more optimistic about the stock market performance (i.e., %Bull) seem to be late to the recession. This type of investor is receiving bad macro news that conflicts with their optimistic beliefs. Therefore, they update their beliefs in the right direction, but not by enough, relative to the rational benchmark. Conversely, as panel (c) shows, pessimistic investors (i.e., %Bear) seem to already reflect macroeconomic contractions.

Interestingly, the pattern of overall sentiment nicely aligns with the strategy’s risk exposure during recessions. As investors slowly incorporate recessionary macroeconomic news, they will have increasingly bearish expectations for the stock market. As a consequence, investors will slowly reduce their demand for the stock market. This means that returns during these periods will be particularly low. Notably, the strategy completely cashes out during these periods—avoiding negative returns—but it returns to the market once sentiment reaches its minimum, around 6 months into a recession (see panel (a) of Figure 5). As economic conditions begin to improve, investors become more bullish. As a result, stock prices increase, thereby generating higher future returns, from which my strategy benefits by increasing risk-taking.

My survey-based evidence is only suggestive and a full investigation is required before definitive explanation of my results can be made. I leave this investigation to future work.

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37 I assume a recession occurs when the recession probability $\pi_t$ exceeds 50%.
38 Consistent with my findings, Amromin and Sharpe (2009) show that household investors have procyclical stock market return expectations.
39 This behavior is known in the psychology literature as conservatism. Barberis et al. (1998) use conservatism to propose a behavioral model of investor sentiment.
6 Conclusions

I have used data on a variety of economic and financial indicators to identify business-cycle turning points in real time. Using these estimates, I have documented a strong feature of the U.S. stock market: time-series reversals in periods of broad economic growth, but times-series momentum during contractions. I then propose a new measure of expected returns to time the market that optimally exploits the state-dependent autocorrelation in returns. This trading strategy earns large alphas (7.4% annualized), increases Sharpe ratios (58% over a buy-and-hold strategy), and generates big utility gains (of around 398 bps) for a mean-variance investor. I document similar profitability for returns on a wide range of characteristic sorted portfolios, international equity markets, and across multiple time periods. Importantly, this predictability is driven largely by the macroeconomic data rather than asset prices.

The investment strategy is contrary to conventional wisdom, because it assumes less risk at the onset of expected bad economic times, suggesting that expected returns are low or negative—not high—during these periods. This rules out standard risk-based explanations for my findings. After cashing out, my strategy aggressively returns to the market shortly afterward (6 months on average), which may be consistent with temporary downward price pressure caused by investors' underreaction to unfavorable shifts in the economic conditions. I find support for this view in surveys of investors’ expectations.

Why don’t investors exploit this likely market inefficiency to the point of eliminating mispricing? A brief answer is that investors are either unaware of this mispricing, or they are unable of taking advantage. If the former, I would expect this mispricing or profit opportunity to disappear/attenuate as investors become aware (e.g., [McLean and Pontiff 2016] [Schwert 2003]). However, some anomalies still persist after being analyzed in the academic literature and traded by practitioners (e.g., [Jegadeesh and Titman 2001]). Alternatively, big commissions, large price impacts, high direct transaction costs (beyond those studied in Section A.2), or institutional frictions could preclude active fund managers of taking advantage of the substantial risk adjusted returns documented here.
References


Appendix

The Appendix consists of two main sections. In Section A I present additional empirical results. In Section B I provide the details of the forecasting model.

A Additional Empirical Results

In this section I present the robustness checks of the main profitability results. Each section is self-contained and can be skip without loss.

A.1 Considering different pricing anomalies and subsample periods

Table A1 shows that the main specification has a positive alpha with respect to a wide range of pricing factors and across different subsample periods. I present results for several combinations of factors including the Fama-French three and five factors and the momentum factor. I also show that it is robust to low-risk anomalies such as the betting-against-beta factor (BAB) of Frazzini and Pedersen (2014). Reassuringly, for the whole 1980-2016 sample period, all alphas are statistically significant and economically large ranging from 5.1% to 7.8%. One might worry that these results are entirely driven by the Great Recession that began in December 2007 and ended in June 2009. To address this concern, I also analyze the performance of the strategy across two subsamples ranging from 1980 to 2000 and from 2001 to 2016, respectively. In both subsamples, the alphas are substantial although in most cases the point estimates are higher in the latter period.

A.2 Leverage, Short-selling, and Transaction Costs

I also show that the strategy produces fairly large benefits even with tight leverage and short-selling constraints. Moreover, transaction costs would need to be incredibly high to make the strategy unprofitable.

Panel (a) of Table A2 shows the distribution of the portfolio weights under different scenarios of leverage and short-selling constraints together with various performance measures. The first row is the benchmark strategy which does not impose any constraints. Noteworthy, the benchmark strategy uses modest levels of short-selling and leverage: the 5% and 95% quintiles of the weight
Table A1. Controlling for Various Pricing Factors and Subsample Analysis

This table reports the alphas from a time-series regressions of the market-timed portfolio on different pricing factors considered in the literature. To time the market I used the main forecast. With respect to the pricing factors I considered the Fama-French three and five factor models (FF3 and FF5, respectively); the momentum factor (Mom); and the betting-against-beta factor (BAB). I also show results for subsample regressions of 15-year periods. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. To facilitate interpretation, all returns are annualized in percent per year by multiplying monthly returns by 12. The estimation sample runs from 1980:M1 to 2016:M12.

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<td>FF3</td>
<td>Fm</td>
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<tr>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>1980-2016</td>
<td>7.43</td>
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<td>7.42</td>
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<td>5.14</td>
<td>5.74</td>
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<td>(2.50)</td>
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<td>(1.82)</td>
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<td>7.37</td>
<td>4.63</td>
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<td>6.00</td>
<td>6.55</td>
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</tr>
<tr>
<td>1980-2000</td>
<td>(2.87)</td>
<td>(2.51)</td>
<td>(2.96)</td>
<td>(1.73)</td>
<td>(2.25)</td>
<td>(2.46)</td>
<td>(2.78)</td>
<td>(2.17)</td>
<td>(2.43)</td>
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<tr>
<td>α:</td>
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<td>8.49</td>
<td>8.31</td>
<td>9.66</td>
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<td>5.91</td>
<td>5.92</td>
<td>7.35</td>
<td>7.51</td>
</tr>
<tr>
<td>2001-2016</td>
<td>(1.98)</td>
<td>(1.95)</td>
<td>(1.79)</td>
<td>(1.95)</td>
<td>(1.99)</td>
<td>(1.60)</td>
<td>(1.63)</td>
<td>(1.71)</td>
<td>(1.79)</td>
</tr>
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</table>

These numbers suggest that the impressive performance of the managed portfolio is not due to unrealistically high levels of short-selling or leverage, but rather to the ability of the forecasting strategy to time the market. To see this, in the second row I preclude the strategy to go short, while in the third and fourth row I add a leverage constraint by imposing a cap on the upper bound of the portfolio weights of 1.5 (50% standard margin requirement) and 1.0 (no-leverage), respectively. Several aspects of these restrictions are noteworthy; here I simply mention a few. First, these constraints reduce substantially trading activity as measured by the mean absolute difference of the monthly weights, $|Δω|$. However, the overall performance of the strategies do not decrease by much. For instance, even under the extreme case of no short-selling and leverage, the strategy earns a significant alpha of 3%, achieves a performance fee of 50 basis points, while the Sharpe ratio essentially does not decrease.

Panel (b) of Table A2 shows that the performance of the strategy survives realistic levels of transaction costs. I report the performance fee that an investor would be willing to pay to have access to the dynamic strategy under various trading costs. Specifically, I consider three different values equal to 1, 10, and 20 basis points. To put these numbers into perspective, Fleming, Kirby, and Ostdiek (2003) estimate a cost of around 1 basis point of the traded value in the futures market.

The last line computes the transaction cost in basis points needed to drive the performance fee to

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40 The 1% and 99% quintiles are -1.6 and 2.0, respectively.
Panel (a): Leverage and short-selling constraints

<table>
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<tr>
<th>weight limits</th>
<th>Distribution of Weights $\omega_t$</th>
<th>Sharpe $\alpha$</th>
<th>Sharpe increase $\Delta(\text{bps})$</th>
<th>Break</th>
</tr>
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<td>$[-, -]$</td>
<td>0.27 0.52 0.89 1.16 1.68 10.76 7.40</td>
<td>0.69 58.32</td>
<td>396.33 393.57 364.17 331.50 123bps</td>
<td>None</td>
</tr>
<tr>
<td>[0, -]</td>
<td>0.00 0.52 0.89 1.16 1.68 9.45 4.91</td>
<td>0.70 60.35</td>
<td>289.05 286.51 263.64 238.21 115bps</td>
<td>1bps</td>
</tr>
<tr>
<td>[0, 1.5]</td>
<td>0.18 0.52 0.89 1.16 1.50 8.94 4.55</td>
<td>0.70 59.97</td>
<td>244.55 242.35 222.55 200.53 113bps</td>
<td>10bps</td>
</tr>
<tr>
<td>[0, 1.0]</td>
<td>0.12 0.52 0.89 1.00 1.00 7.24 3.41</td>
<td>0.67 53.86</td>
<td>92.47 91.01 77.85 63.22 65bps</td>
<td>20bps</td>
</tr>
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</table>

Table A2. Leverage, Short-selling, and Transaction Costs

This table reports various performance measures for several alternative constraints on the market-timed portfolio and for different transaction costs. The alternative constraints consist of restricting the risk exposure to be above zero (i.e., no short-selling), below 1 (i.e., no leverage), or below 1.5 (50% standard margin requirement). With respect to the transaction costs, I consider 1, 10, and 20 basis points. I also report the trading costs needed to drive the performance fee to zero. To facilitate interpretation, all returns are annualized in percent per year by multiplying monthly returns by 12. The estimation sample runs from 1980:M1 to 2016:M12.

zero. Overall, the results of panel (b) suggest that transaction costs would need to be incredible high to eliminate the market timing gains.

A.3 Stale information about the economic outlook

I have implemented the strategy by using the probability of being in a period of broad economic contraction $\hat{\pi}_{t|t}$. Furthermore, I have argued that the estimation approach is able to identify in a timely fashion changes in the economic outlook. Next, I introduce stale information pertaining to the economic conditions and ask how stale can it be for the results to hold. To this end, I modify the aggregate predictive density in (9) as follows:

$$p(r_{t+1}^e|\Theta, r_{1:t}^e) = (1 - \hat{\pi}_{t|t-h}) p(r_{t+1}^e|\Theta, r_{1,t}^e, S_t = 0) + \hat{\pi}_{t|t-h} p(r_{t+1}^e|\Theta, r_{1,t}^e, S_t = 1).$$

(27)

I introduce sticky information about the economic states by considering information up to time $t - h$ at the time of doing the $t$ forecast of the regimes.

Figure A1 presents the results. Panel (a) shows the market alpha of the strategy for different lag $h$-periods used to compute the forecasting weights $\hat{\pi}_{t|t-h}$. The main takeaway is that the strategy remains profitable even if we impose stale information about the cycle: The alpha is positive up to the fifth month lag. However, it sharply declines beyond the second month. To better understand where the loss in profitability is coming from, panel (b) and (c) of Figure A1 plot the alpha obtained
in NBER expansions and recessions, respectively. As expected, most of the action happens in recessions. The alpha in recessions is substantial up-to the second month gap, but after the third month it turns negative and big (in absolute terms) driving to zero the overall performance of the strategy (see panel (a)). This last result highlights the importance of identifying the regimes in a timely fashion.

### A.4 Importance of the two key model ingredients.

To assess the economic relevance of the two key assumed model ingredients, I turn-off each one of them at a time and re-estimate the model. In column (1) of Table A3, I show the performance of a model that ignores the first assumed key property of expected returns by setting to zero the correlation between the unexpected return and news to the expected return (i.e., $\rho_{\epsilon_\mu,r} = 0$).

Moreover, in column (2) I show the results for a model that also shuts down the second assumed key property of returns by turning-off the state-dependent variability of expected returns (i.e., $\rho_{\epsilon_\lambda,r} = 0$ and $\phi_1 = \phi_2$). In both cases we see that the profitability disappears: The alphas are small and

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41 In essence, column (3) of Table 2 shows the results of a model that shuts-down the second key property of expected returns by setting $\phi_1 = \phi_2$ (and predicting using only the predictive density associated with the low $\phi$). It is not exactly the same, since I would need to set $\phi_1 = \phi_2$ and re-estimate the model again. In practice, it turns out that I get similar results using both methods; therefore, for the sake of conciseness, I do not report the results obtained under the latter.
Table A3. Performance Evaluation after Restricting the Information Set

This table presents results of the economic and statistical significance of the market timing strategy. I consider two restricted versions of the model. All models are estimated out-of-sample on recursively increasing samples of real time data. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. To facilitate interpretation, all returns are annualized in percent per year by multiplying monthly returns by 12. The forecast evaluation period runs from 1980:M1 to 2016:M12.

statistically insignificant. The Sharpe ratios are lower than the Sharpe ratios of the buy-and-hold strategy. Furthermore, both the performance fee and the \( R^2_{OOS} \) statistics are negative. These results stress the economic importance of the two key model ingredients. I further assess their statistical significance by computing the log marginal data density, \( \ln p(Y) \), for each specification. The restricted models are strongly dominated with log posterior odds of 6 and 197 in favor of the unrestricted model, respectively (the last row of Table 1 shows \( \ln p(Y) \) for the main specification).

A.5 Four additional market-timing test.

In this section I present four additional timing-related performance measures. The first measure is based on the quadratic regression of Treynor and Mazuy (1966):

\[
r_{p,t+1} = \gamma_0 + \gamma_1 r_{m,t+1}^e + \gamma_2 \left(r_{m,t+1}^e\right)^2 + \epsilon_{t+1},
\]

\[ (28) \]
where the coefficient $\gamma_2$ measures the return timing performance. As shown in panel (a) of Table A4, the $\gamma_2$ coefficient associate with the main predictor is significantly positive. Note that with the exception of the Low-Low forecast, the $\gamma_2$ coefficients associated with the alternative and restricted models are economically small and statistically insignificant. As expected, given that the quadratic regression term $(r_{m,t+1}^e)^2$ captures timing abilities, the abnormal return $\gamma_0$ for the main specification is small and insignificant. The second measure is due to Henriksson and Merton (1981) and replaces the quadratic term $(r_{m,t+1}^e)^2$ with $\max(-r_{m,t+1}^e, 0)$:

$$r_{p,t+1} = \gamma_0 + \gamma_1 r_{m,t+1}^e + \gamma_2 \max(-r_{m,t+1}^e, 0) + \epsilon_{t+1}$$  \hspace{1cm} (29)$$

Similarly, a positive $\gamma_2$ suggests market timing ability. Relative to the measure of Treynor and Mazuy (1966), the emphasis of this market timing measure is on how well it predicts during market downturns. Panel (b) of Table A4 shows the results for this specification. The positive $\gamma_2$ under the main specification provides further supporting evidence of the market-timing abilities during downturns.

The third market-timing test is the manipulation-proof performance measure (hereafter MPPM) of Goetzmann et al. (2007). Traditional measures (e.g., alphas, appraisal ratios, Sharpe ratios) are vulnerable because they can be manipulated. The MPPM statistic (by design) is robust to manipulations and can be written as:

$$MPPM = \frac{1}{(1-\rho)\Delta t} \ln \left( \frac{1 + \sum_{t=1}^{T} \left[ \frac{1 + r_{p,t}}{1 + r_{f,t}} \right]^{1-\rho}}{T} \right)$$  \hspace{1cm} (30)$$

where $r_{f,t}$ denotes the risk-free rate and $r_{p,t}$ denotes the per-period (not annualized) portfolio return. I select the coefficient $\rho (\approx 2.7)$ such that holding the market is optimal for an uninformed manager. Panel (c) of Table A4 shows that the MPPM statistic for the main specification is equal to 6.7% and can be interpreted as the annualized continuously compounded excess return certainty.

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43 After controlling for $(r_{m,t+1}^e)^2$, a positive $\gamma_0$ would suggest superior stock selection which is not considered in this paper.

44 Portfolio managers can use dynamic strategies or derivatives to alter the distribution of returns to artificially enhance any particular performance measure or deliberately generate measurement errors to his benefit. For details and examples see Goetzmann et al. (2007).

45 Specifically, I set $\rho = \frac{\ln[\hat{r}(1+\hat{r}_m) - \ln(1+\hat{r}_f)]}{\text{var}(1+\hat{r}_m)}$ where $\hat{r}_m$ and $\hat{r}_f$ are the unconditional mean of the market and risk-free rate.
Table A4. Four additional market-timing test.
This table presents four market-timing test for the different specifications considered and described in the main text. Panel (a) and panel (b) show results for the timing measures of Treynor and Mazuy (1966) and Henriksson and Merton (1981). Panel (c) shows results for the Manipulation-Proof Measures of Performance of Goetzmann et al. (2007), while panel (d) presents the correlation between the portfolio weights at time $t$ and the $t + 1$ realized excess returns. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. The out-of-sample test runs from 1980:M1 to 2016:M12.

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<td>Rec.</td>
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<td>Low–High</td>
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<td>$S_t = 0$</td>
<td>$S_t = 1$</td>
</tr>
</tbody>
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Panel (a): Treynor and Mazuy (1966)

$$r_{p,t+1} = \gamma_0 + \gamma_1 r^{e}_{m,t+1} + \gamma_2 (r^{e}_{m,t+1})^2 + \epsilon_{t+1}$$

| $\gamma_0$     | 0.15          | -1.44              | 4.30            | 3.81          | 2.73          |
|                | (0.06)        | (-0.68)            | (1.49)          | (1.31)        | (1.92)        |
| $\gamma_1$     | 0.57          | 0.79               | 0.40            | 0.35          | 0.85          |
|                | (11.84)       | (21.01)            | (7.73)          | (6.79)        | (33.59)       |
| $\gamma_2$     | 2.50          | 1.52               | 0.10            | -0.61         | -0.93         |
|                | (5.57)        | (4.30)             | (0.20)          | (-1.26)       | (-3.91)       |

Panel (b): Henriksson and Merton (1981)

$$r_{p,t+1} = \gamma_0 + \gamma_1 r^{e}_{m,t+1} + \gamma_2 \max(0, r^{e}_{m,t+1}) + \epsilon_{t+1}$$

| $\gamma_0$     | -7.39         | -4.47              | 0.67            | 2.27          | 3.08          |
|                | (-1.94)       | (-1.49)            | (0.17)          | (0.55)        | (1.52)        |
| $\gamma_1$     | 0.85          | 0.92               | 0.50            | 0.37          | 0.81          |
|                | (9.28)        | (12.74)            | (5.11)          | (3.79)        | (16.59)       |
| $\gamma_2$     | 0.68          | 0.34               | 0.18            | -0.01         | -0.14         |
|                | (4.86)        | (3.08)             | (1.24)          | (-0.04)       | (-1.84)       |

Panel (c): Manipulation-Proof Measures of Performance of Goetzmann et al. (2007)

**MPPM**

6.71  4.15  3.76  0.95  2.31

Panel (d): Correlation

$$\text{Corr}(\omega_t, r^{e}_{m,t+1})$$

0.14  0.09  0.06  0.02  -0.02

The fourth market-timing test is more intuitive and straightforward. A successful market-timing investor would increase her exposure on the risky asset when future returns are high and will decrease it when future returns are low. Hence, if the investor is successful, the correlation between the forecasting weights $\omega_t$ and the $t + 1$ realized returns $r^{e}_{m,t+1}$ is expected to be positive. Indeed, panel (d) Table A4 shows a positive correlations for the main specification.

equivalent of the portfolio. Noteworthy, the MPPM statistic under the main specification is the highest relative to the other specifications.
Table A5. Characteristic-sorted portfolio returns

This table presents results of the economic and statistical significance of the Characteristic-sorted portfolio excess returns. I consider only the results for the main specification estimated out-of-sample using an expanding window. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. To facilitate interpretation, all returns are annualized in percent per year by multiplying monthly returns by 12. The forecast evaluation period runs from 1980:M1 to 2016:M12.

Overall, for each of these different market-timing measures, I find that my market-timing strategy consistently beats the buy-and-hold portfolio as well as the alternative model specifications.

A.6 Characteristic-sorted portfolio returns

Table A5 shows the results when, instead of timing the excess market return, I time characteristic-sorted portfolio returns in excess of the risk free rate. Specifically, I consider the first, third and fifth quintile on stock book-to-market ratios, market capitalization, and momentum portfolios from Ken French’s website. I show results only for the main specification. As expected, the strategy works equally well for these returns. The strategy increases alphas (with respect to FF3 and Momentum), Sharpe ratios, and generates big utility gains for a mean-variance investor with risk aversion coefficient equal to 3. The last row also reports big out-of-sample $R^2$ above 1% per month.

A.7 Anomaly return spreads.

So far I have formed market-timed portfolios on returns that have a positive exposure to the market. In this section, I analyze the performance of my strategy on return spreads that essentially hedge market risk. In particular, I consider the size factor (SmB), value factor (HmL), momentum (Mom), profitability factor (RmW), and investment factor (CmA). All returns are at the monthly frequency and come from Kenneth French’s website.

Similar as in Section 4.1, I form portfolios by timing the market in which the managed portfolio
Table A6. Anomaly return spreads.
This table presents results of the economic and statistical significance of the different anomaly return spreads. I consider only the results for the main specification estimated out-of-sample using an expanding window. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. To facilitate interpretation, all returns are annualized in percent per year by multiplying monthly returns by 12. The forecast evaluation period runs from 1980:M1 to 2016:M12.

\[
\hat{f}_{t+1} = \omega_t \times f_{t+1} \tag{31}
\]

where \( \omega_t = c \times E_t[f_{t+1}] \) and \( f_{t+1} \) is the buy-and-hold pricing factor. The expected factor return \( E_t[f_{t+1}] \) is computed as in Section 2. Table A6 shows the results. For most of the pricing factors the alphas are economically small and statistically insignificant. The only exception is on the momentum factor, for which the market-timed momentum factor has an annualized alpha of around 6% (46% Sharpe increase), and a beta of 0.60. If the forecasting strategy captures movements in the aggregate market well, it is perhaps unsurprising that the strategy does not work well for return spreads that are design to hedge market risk.
A.8 Extended sample period.

Above I report out-of-sample results based on a 1985:01 to 2016:12 sample split. In this section, I demonstrate the robustness of the out-of-sample forecasts to an earlier sample period. Ideally, I should predict recursively the economic states using real-time data as I did in the main text. However, most of the macro data available to use start in 1960. Thus, the earliest that I can start the out-of-sample forecasts using this approach is from 1980 onwards (considering that we need approximately 20 years of data as a training sample).

To deal with is issue, I make use of the ex-post NBER recession indicator. This allows me to extend the forecasting sample period as early as 1960, which uses around 30 years of data as training sample (from 1930:01 to 1959:12). With this in mind, I modify the aggregate predictive density in (9) as follows:

$$p(r_{t+1}^e | r_{1:t}^e, \Theta, \pi_t) = (1 - I_{rec,t-h}) p(r_{t+1}^e | r_{1:t}^e, \Theta_{S_t=0}, S_t = 0) + I_{rec,t-h} p(r_{t+1}^e | r_{1:t}^e, \Theta_{S_t=1}, S_t = 1) \quad (32)$$

where $I_{rec,t-h}$ is the NBER recession indicator, which takes a value of 1 if period $t-h$ is recessionary and a value of 0 if it is expansionary. Importantly, I introduce stale information about the economic state by lagging the recessionary indicator by $h$-periods at the time of doing the forecasts and form portfolios as before. This exercises simulates the scenario of an investor who needs $h$ periods to identify that she is in a recession. For instance, when $h = 0$ the investor perfectly foresees recessions and expansions, while $h = 3$ means that she needs around 3 months of data to identify business-cycle turning points.

Table A7 shows the results. I draw two important conclusions from these results. First, the strategy is still profitable even under stale information about the cycle. However, the outperformance of the strategies vanishes after the six-month lag. Note that for values of $h$ greater than 6, the alphas are insignificant and small. Similarly, the $R^2_{def}$ statistics are negative. Second, there is strong evidence that the strategy works well for the extended 1960:01 to 2016:12 sample period.

A.9 International Data.

I extend the robustness checks by looking at international data. In particular, the results hold for 9 OECD countries as well as for the Global aggregate index. To this end, I use for each country the
Lag in the forecasting weights relative to the NBER Recessionary Index

<table>
<thead>
<tr>
<th>Time Period 1960 - 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
</tr>
<tr>
<td>6.10</td>
</tr>
<tr>
<td>(3.20)</td>
</tr>
<tr>
<td>Original Sharpe</td>
</tr>
<tr>
<td>0.33</td>
</tr>
<tr>
<td>0.33</td>
</tr>
<tr>
<td>New Sharpe</td>
</tr>
<tr>
<td>0.52</td>
</tr>
<tr>
<td>0.46</td>
</tr>
<tr>
<td>$R^2_{\text{OOS}}$ (%)</td>
</tr>
<tr>
<td>1.94***</td>
</tr>
<tr>
<td>1.48***</td>
</tr>
<tr>
<td>1.60***</td>
</tr>
<tr>
<td>1.59***</td>
</tr>
<tr>
<td>1.11**</td>
</tr>
<tr>
<td>-0.41</td>
</tr>
<tr>
<td>-0.52</td>
</tr>
</tbody>
</table>

**Table A7. Market timing performance for lag Information about the economic states**

This table shows results by introducing stale information pertaining to the economic conditions to the forecasting strategy. The columns correspond to the induced lag in months in the NBER recession dummy. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. The sample runs from 1960:01 to 2016:12. The training sample runs from 1930:01 to 1959:12.

modified predictive density in 52 by replacing the NBER recessionary index with the corresponding OECD based recession indicators and lag the indicator accordingly.

Table A8 shows the results. On average, market-timing portfolios produce big alphas with respect to the buy-and-hold strategy for lags in the forecasting weights of around 6 months. That is, international investors will find beneficial to time the market using my strategy if they are able to identify business-cycle turning points with a lag smaller than 7 months.

**A.10 What about cash-flow risk?**

I have argued that variations in risk premia are the main drivers of the rich dynamics of excess returns observed during recessions. Yet, expected future dividend growth rates could be an alternative mechanism driving the pattern. In this section I analyze this possibility.

I start by extending the VAR analysis of Campbell (1991) and allow for a state-dependent decomposition of shocks to market returns. Specifically, I model the agents’ expectations by a Markov-switching VAR (MS-VAR):

$$Z_{t+1} = A_0, s_{t+1} + A_1, s_{t+1} Z_t + u_{t+1}, \quad u_{t+1} = V s_{t+1} \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, I),$$

where the state variable $S_{t+1}$ controls the regime that is in place at time $t+1$, which evolves according to a Markov chain. Given the evolution of the state variables, $Z_{t+1}$, the unexpected
Table A8. Market timing performance: International evidence
This table shows results across different OECD countries. The columns correspond to the induced lag in months in the OECD recession dummy. Standard errors adjust for heteroskedasticity and autocorrelation. I report the t-statistics in parentheses. The sample runs from 1995:01 to 2016:12. The training sample runs from 1970:01 to 1994:12.

\[ RN_{t+1} = r_{t+1} - E_t[r_{t+1}] \approx (E_{t+1} - E_t) \sum_{j=1}^{\kappa} \kappa^{-1} \Delta d_{t+j} - (E_{t+1} - E_t) \sum_{j=2}^{\kappa} \kappa^{j-1} r_{t+j} \]

\[ = CFN_{t+1} - DRN_{t+1} \]
where $\Delta d_{t+1}$ is the log of dividend growth, $r_{t+1}$ is the log aggregate market return, and $\kappa$ is a log-linearization parameter less than one. Hence, revisions in returns, $RN_{t+1}$, are due to revisions in the expectations about current and future dividend growth, $CFN_{t+1}$, or revisions in future expected returns $DRN_{t+1}$. In the presence of regime changes, the updates in beliefs can be easily computed by applying the methods developed in [Bianchi (2016)].

My primary MS-VAR specification is at the monthly frequency and includes five state variables: the log aggregate market return, the price-to-earning ratio, the book-to-market ratio, the term spread, and the default yield spread. All of these variables are common predictors in the forecasting literature (see for example [Rapach et al. (2013)]). Furthermore, I exogenously specify the regimes, $S_{t+1}$, and assume that they evolve according to the real-time recessionary probabilities $\pi_t$ discussed in Section 2.1.1. I assume that the economy is expected to be in a recession when the probability is expected to be above 50% (i.e., $\pi_{t+1|t} > 0.5$). Finally, I set the constant $\kappa$ to 0.997, which is a standard value used in the literature and corresponds to an annual dividend-price ratio of 4%.

The first columns of Table A9 present the state-dependent variance decomposition of market returns. These results are based on the 1985-2016 sample and to facilitate comparison across different regimes, I only report the posterior median values. The first row shows the overall annualized volatility of the return news, while the second to fourth decompose its variance into three exhaustive components: the variance of cash-flow news, discount rate news, and the covariance between these two news. As expected, the conditional volatility of returns news is higher in expected recessions relative to expansions (90% and 50%, respectively.) Interesting distinctions arise in the decomposition of return shocks across regimes. In expected expansions, cash-flow news variation is slightly more important and accounts for 42% of the return news variance, whereas discount rate variation accounts for 35%. The covariance term between CFN and DRN is negative and accounts for 23% of the overall variation in RN.

Importantly, in expected recessions almost all the variation in return news is accounted for by

---

Note: The findings are robust to several alternative specifications. The results hold if I consider other predictors used in [Goyal and Welch (2008)], such as the log dividend-price ratio, log dividend yield, excess stock return volatility, net equity expansion, or the default return spread. They are also robust to alternative rules to define the probability based recessionary dummy variable. For instance, I also consider that a recession starts if $\pi_{t+1|t}$ is above 60% and ends once $\pi_{t+1|t}$ is below 30%. Furthermore, the results remain for different reasonable values of the log-linearization parameter $\kappa$ ranging from 0.98 to 0.999, which correspond to annual dividend-price ratio of 27% and 1.2%, respectively.
### Table A9. Market Return Variance Decompositions

This table shows the state-dependent decomposition of the log market return news (RN) into its cash flow news (CFN) and discount rate news (DRN) components. To condition on the state of the economy I consider both, the real-time recessionary probabilities computed in Section 2.1.1, and the NBER recession dummy. The sample for the former runs from 1985:01 to 2016:12, whereas the sample for the latter runs from from 1930:01 to 2016:12.

Fluctuations in discount rates news: variations in DRN accounts for 95% of the overall variation in RN. Surprisingly, the covariance term between CFN and DRN turns positive, suggesting that low expected cash flows are associated with low discount rate news. That is, while in an expected recession, the aggregate risk premium (or discount rates) tends to decrease after lower-than-expected cash-flow realizations. Finally, in the last row I report the in-sample return predictability across economic states based on the estimation of the MS-VAR. Consistent with the results of the forecasting model in Section 3, return predictability is higher in recessions and hence reinforce the prominent role that discount rate variations play for realized return variations during economic contractions.

Columns of (5) to (6) in Table A9 show results using the NBER-defined recession dates for the extended sample period that goes from 1930 to 2016. The role of discount rate variations during recessions is even stronger. In this scenario, discount rate news accounts for 114% of the return variation, cash-flow news accounts for 19%, and the positive covariation term between these two components accounts for -34%. Similarly, expected returns explain more of the variation in realized returns as measured by the in-sample R-squared. Lastly, as shown in the last three columns, these results remain largely unchanged if I introduce a three month lag in the NBER index, which is consistent with the time that an investor appears to need in order to identify these economic states.

<table>
<thead>
<tr>
<th></th>
<th>Real-time recessionary states</th>
<th>NBER recessionary states</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \pi_{t+1</td>
<td>t} &gt; 0.5 )</td>
</tr>
<tr>
<td>Tot. Exp. Rec.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sqrt{\text{var}(RN)} \times 1200% )</td>
<td>53.2 50.8 90.6</td>
<td>63.7 53.1 98.8</td>
</tr>
<tr>
<td>Variance Decomposition as fraction of ( \text{var}(RN) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{var}(CFN) \times 100% )</td>
<td>46.5 41.9 28.2</td>
<td>20.9 25.1 19.2</td>
</tr>
<tr>
<td>( \text{var}(DRN) \times 100% )</td>
<td>41.1 35.1 94.9</td>
<td>74.9 49.9 114.8</td>
</tr>
<tr>
<td>(-2\text{Cov}(CFN, DRN) \times 100% )</td>
<td>11.9 21.9 -27.7</td>
<td>3.6 22.3 -34.0</td>
</tr>
<tr>
<td>In-sample return predictability ( R^2 )</td>
<td>2.3 1.4 18.7</td>
<td>2.9 1.4 8.6</td>
</tr>
</tbody>
</table>
in real time.

Based on the MS-VAR evidence, discount rates become more important relative to cash flows in recessions, suggesting that my forecasting strategy is most likely capturing discount rate variations. Moreover, given that recessions are by definition periods of low realization of fundamentals, the positive covariance between cash-flow news and discount rate news in these bad times strengthens the central finding of this paper: At the onset of expected contractions, risk premia is low or negative.
B Forecasting Model: Details and Proofs

B.1 Data

The dataset consists of macroeconomic and financial variables measured at the monthly and quarterly frequency. All the series start at different periods but all end in 2016:M1

<table>
<thead>
<tr>
<th>Macro Variables</th>
<th>Financial Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial Production Index, 1962:M10</td>
<td>Dividend-price ratios, 1959:M1</td>
</tr>
<tr>
<td>GDP, 1965:Q3</td>
<td>Earnings-price ratio, 1959:M1</td>
</tr>
<tr>
<td>Real Personal Income Excluding Current Transfer Receipts, 1959:M2</td>
<td>10-2 Year Treasury Yield Spread, 1959:M1</td>
</tr>
<tr>
<td>Aggregate Weekly Hours Worked, 1971:M8</td>
<td>10-Year Treasury Yield minus Federal Funds Rate, 1962:M1</td>
</tr>
<tr>
<td>Personal Consumption Expenditure 1965:Q4</td>
<td>Credit Spread Moody AAA-BAA, 1959:M1</td>
</tr>
<tr>
<td>Residential Gross Private Domestic Investment 1965:Q4</td>
<td>VIX, 1990:M1</td>
</tr>
<tr>
<td>CPI 1989:M12</td>
<td>Debit balances in margin accounts at broker/dealers, 1959:M2</td>
</tr>
</tbody>
</table>

B.2 State-Space Representations

In this section I describe the state-space representation of the forecasting model.

B.2.1 State-space representation of the forecasting model weights. Below I describe the state-space representation of the forecasting model weights. The representation consists of a
measurement equation:

\[ y_{t+1} = A_{t+1} (H_0 + H_1 x_{t+1} + \nu_{t+1}) \quad \text{with} \quad \nu_{t+1} \sim N(0, R); \]  

(35)

where \( A_{t+1} \) is a selection matrix that accounts for changes in the vector of observables \( y_{t+1} \). The state-transition equation is:

\[ x_{t+1} = F_0 (S_t, S_{t+1}) + F_1 (S_{t+1}) x_{t+1} + \omega_{t+1} \quad \text{with} \quad \omega_{t+1} \sim N(0, Q(S_{t+1})) \]  

(36)

I will describe each in turn.

**Measurement equation.** The measurement equation provides the link between the vector of observables \( y_{t+1} \) and the vector of state variables, \( x_{t+1} \), this link is determined by the assumed processes in equation (1):

\[ y_{i,t+1} = \gamma_i z_{t+1} + e_{i,t+1} \quad \text{for} \quad i = 1, \ldots, N_{\text{Macro}}. \]  

(37)

To facilitate the model estimation, I further specify \( y^*_i,t+1 = y_{i,t+1} - \psi_i y_{i,t} \). Hence, equation (37) simplifies to

\[ y^*_{i,t+1} = \gamma_i z_{t+1} - \gamma_i \psi_i z_t + \sigma_i e_{i,t} \quad \text{for} \quad i = 1, \ldots, N_{\text{Macro}} \]

Conditional on the parameters, the vector of observables is given by \( y_{t+1} = [y^*_{1,t+1}, y^*_{2,qt,t+1}]' \). For the sake of exposition, I only consider two observables (i.e., \( N_{\text{Macro}} = 2 \)), and I further assume that the first variable is available at the monthly frequency (e.g., Industrial production growth), while the second variable is available only at the quarterly frequency (e.g., GDP growth). The link between the quarterly time-aggregated series with the model-implied monthly frequency is given by the following tend function described in the main text:

\[ y^*_{2,qt,t+1} = \sum_{j=1}^{5} \frac{3 - |j - 3|}{3} y_{2,t+2-j} \]  

(38)

such that the monthly model variables \( y_{2,t+2-j} \) time-aggregate to the observed quarterly growth rates \( y^*_{2,qt,t+1} \) (to simplify the notation I further omitted the \( o \)-subscript used for the observable variables in the main text).
The selection matrix $A_{t+1}$ and the vector of observables $y_{t+1}$ are time-dependent. Specifically, we have:

- If $t + 1$ is the last month of the quarter:
  \[
  y_{t+1} = \begin{bmatrix} y_{1,t+1}^* \\ y_{2,t+1}^*,qrt \end{bmatrix}_{2\times 1} \\
  A_{t+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}
  \]

- If $t + 1$ is the first or second month of the quarter:
  \[
  y_{t+1} = \begin{bmatrix} y_{1,t+1}^* \end{bmatrix}_{2\times 1} \\
  A_{t+1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}
  \]

To define the matrices associated with the measurement equation (i.e., $H_0$, $H_1$, and $R$), I first specify the state vector $x_{t+1}$ as

\[
x_{t+1} = \begin{bmatrix} z_{t+1} & z_t & z_{t-1} & z_{t-2} & z_{t-3} & z_{t-4} & \sigma_1 \epsilon_{1,t+1} & \sigma_2 \epsilon_{2,t} & \sigma_2 \epsilon_{2,t-1} & \sigma_2 \epsilon_{2,t-2} & \sigma_2 \epsilon_{2,t-3} & \sigma_2 \epsilon_{2,t-4} \end{bmatrix}'_{12\times 1}
\]

Given the state vector $x_{t+1}$, it follows that

\[
H_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times 1} \\
R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}
\]

\[
H_1 = \begin{bmatrix} \gamma_1 & -\gamma_1 \psi_1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} \gamma_2 & \frac{2}{3} \gamma_2 - \frac{1}{3} \gamma_2 \psi_2 & \frac{2}{3} \gamma_2 - \frac{2}{3} \gamma_2 \psi_2 & \frac{2}{3} \gamma_2 - \frac{2}{3} \gamma_2 \psi_2 & \frac{1}{3} \gamma_2 - \frac{2}{3} \gamma_2 \psi_2 & -\frac{1}{3} \gamma_2 \psi_2 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}_{2\times 12}
\]

**State-transition equation.** The state-transition equation provides the law of motion of the state variables in the model, which evolve according to

\[
z_{t+1} = \mu_z(S_{t+1}) + \phi_z(S_{t+1}) (z_t - \mu_z(S_t)) + \sigma_z(S_{t+1}) \epsilon_{z,t+1}, \tag{39}
\]
Given $x_{t+1}$, it follows that

$$
F_0(S_t, S_{t+1}) = \begin{bmatrix}
\mu_z(S_{t+1}) - \phi_z(S_{t+1})\mu_z(S_t) \\
O_{11 \times 1}
\end{bmatrix}_{12 \times 1},
$$

$$
F_1(S_{t+1}) = \begin{bmatrix}
\phi_z(S_{t+1}) & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}_{O_{6 \times 6}}.
$$

And $Q(S_{t+1})$ is a sparse $12 \times 12$ matrix with the following elements in the $ij$-th position:

$$
Q^{[1,1]}(S_{t+1}) = \sigma_{\varepsilon_t}^2, \quad Q^{[7,7]}(S_{t+1}) = \sigma_1^2, \quad Q^{[8,8]}(S_{t+1}) = \sigma_2^2.
$$

**B.2.2 State-space representation of the process for the excess market return.** Below I describe the state-space representation of the process for the excess market return. In the main text I defined:

$$
r_{t+1} = \mu_t + \varepsilon_{r,t+1},
$$

$$
\mu_{t+1} = \mu_0 + \rho(\mu_t - \mu_0) + \varepsilon_{\mu,t+1},
$$

with $(\varepsilon_{r,t+1}, \varepsilon_{\mu,t+1})' \sim iidN(0, \Sigma(S_t))$, with

$$
\Sigma(S_t) = \begin{bmatrix}
\sigma_{\varepsilon_{\tau}}^2(S_t) & \sigma_{\varepsilon_{\tau}\lambda}(S_t) \\
\sigma_{\varepsilon_{\tau}\lambda}(S_t) & \sigma_{\varepsilon_{\lambda}}^2(S_t)
\end{bmatrix}.
$$

To write the state-space model, I will use the same notation as in equations \((35)\) and \((36)\).

**Measurement equation.** For each time period $t + 1$, the vector of observables and the selection matrix are $y_{t+1} = r_{t+1}$ and $A_{t+1} = 1$, respectively. I write the rest of the matrices as follows:

$$
x_{t+1} = \begin{bmatrix}
\mu_{t+1} & \mu & \sigma_{\varepsilon_r}(S_t)\varepsilon_{r,t+1}
\end{bmatrix}_{3 \times 1}', \quad H_0 = \begin{bmatrix}0\end{bmatrix}_{1 \times 1}, \quad H_1 = \begin{bmatrix}0 & 1 & 1\end{bmatrix}_{1 \times 3}, \quad R = \begin{bmatrix}0\end{bmatrix}_{1 \times 1}.
**State-transition equation.** Given the State vector $x_{t+1}$, the matrices associated with the state-transition equation are given by

\[
F_0 = \begin{bmatrix}
\mu_0(1 - \rho) \\
0 \\
0
\end{bmatrix}_{3 \times 1}, \quad F_1 = \begin{bmatrix}
\rho & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}_{3 \times 3}, \quad Q(S_t) = \begin{bmatrix}
\sigma_{\xi_\tau}(S_t) & 0 & \sigma_{\xi_\lambda}(S_t) \\
\sigma_{\epsilon_\tau}(S_t) & 0 & 0 \\
0 & \sigma_{\xi_\lambda}(S_t) & 0
\end{bmatrix}_{3 \times 3}.
\]

**B.3 Posterior Inference**

**B.4 Posterior Inference: Forecasting model weights**

To estimate the model associated with the forecasting weights described in Section B.2.1, I use the Bayesian Gibbs sampling approach described in [Kim and Nelson (1998)](https://doi.org/10.1086/208474). The assumed prior distribution is quite diffuse, and it is similar to the one used in [Kim et al. (1999)](https://doi.org/10.1080/0022458080237959). The main output of the estimation is the posterior distribution of the economic states $S_t$. The posterior mean of $S_t$ represents the real-time recessionary probabilities, $\pi_t$. Importantly, these probabilities correspond to the dynamic forecasting weights of the predictive model described in Section 2.1.2.

**B.5 Posterior Inference: Process for excess returns**

The state-space representation described in Section B.2.2 is linear conditional on the economic states $S_t$ (estimated in the previous stage, Section B.4). Therefore, for posterior inference of the aggregate predictive model, I can use a standard Metropolis-within-Gibbs sampler.

**Algorithm.** Let $\Theta$ denote the parameter vector and $\mu_{1:T}$, be the sequence of hidden states. The Metropolis-within-Gibbs algorithm involves iteratively sampling from two conditional posterior distributions. The main steps of the sampler are:

1. Initialize $\Theta^{(0)}$ at some value.
2. Draw $\mu_{1:T}^{(s)}$ conditional on $\Theta^{(s-1)}$ and the vector of observables $y_{1:T}$. I obtain draws of $\mu_{1:T}^{(s)}$ by applying the simulation smoother of [Durbin and Koopman (2012)](https://doi.org/10.1093/aje/84.3.207).

\[^{47}\text{For a textbook description of the Gibbs sampler see Chapter 10 of Kim, Nelson et al. (1999).}\]
3. Draw $\Theta^{(s)}$ conditional on $\mu_{1:T}^{(s)}$ and the vector of observables $y_{1:T}$. I am using a Random-Walk Metropolis-Hastings algorithm to generate draws of the model parameters, $\Theta$, from their posterior distribution in (13). Furthermore, I assume that the proposal distribution is $N(\Theta^{(s-1)}, c \Omega)$. for $s = 1, \ldots, N_{\text{sim}}$. For the covariance matrix of the proposal distribution, $\Omega$, I follow Schorfheide (2005) and use the negative inverse of the posterior density Hessian evaluated at the posterior mode $\hat{\Theta}$. I targeted an acceptance rate of approximately 30 percent by adequately modifying the scaling factor $c$. I initialize the algorithm by selecting $\Theta^{(0)}$ to be the posterior mode $\hat{\Theta}$ (estimated using standard maximization algorithms) and generated 25,000 draws (i.e., $N_{\text{sim}} = 25,000$) from the posterior distribution. Finally, I set the burn in period at 15,000 draws.

B.6 Identification

In this section I establish the identification results of the predictive model. I will first show that the model is not identified if we do not impose any restriction on $\Sigma(S_t)$. After establishing this result, I will prove that fixing $\sigma_r^2$ and setting $\rho_{\epsilon,\mu,r}$ to be equal across regimes achieves identification as described in Section 2.2.

A simple way for checking identification is to re-writing the state-space into an ARMA(1,1) process, where we know that the unknown model parameters can be consistently estimated provided that the roots of the MA component are normalized to lie on or outside the unit circle, and are different from the roots of the AR component, assuming that these roots lie also outside the unit circle (see Griliches, Engle, Intriligator, and McFadden (1983) for instance). In this section, I assume that the ARMA(1,1) model is identify, that is, a change in any parameter associated with the ARMA(1,1) would imply a distinct probability distribution for the aggregate market return $\{r_t^e\}_{t=1}^T$ and then I compare them with the parameters implied by the state-space representation.
ARMA(1,1) representation. To re-write the model into an ARMA(1,1) process consider the following state-space model

\[ r_{t+1}^e = \mu_{t+1} + \varepsilon_{r,t+1}, \]
\[ \mu_{t+1} = \rho(S_t)\mu_t + \varepsilon_{\mu,t+1}, \]
\[ (\varepsilon_{r,t+1}, \varepsilon_{\mu,t+1})' \sim iid N(0, \Sigma(S_t)) \quad \text{with} \quad \Sigma(S_t) = \begin{bmatrix} \sigma^2_{\varepsilon_r}(S_t) & \sigma^2_{\varepsilon_{\mu}}(S_t) \\ \sigma^2_{\varepsilon_{\mu}}(S_t) & \sigma^2_{\varepsilon_{\mu}}(S_t) \end{bmatrix} \] (B.1)

where to simplify the subsequent notation I set the constant terms to zero, which can be identify from the conditional mean of \( r_{t+1}^e \). I also modified slightly the timing in the measurement equation, in order to simplify the notation of the ARMA(1,1) process. However, this change in the timing does not affect the identification issues of the model that this section addresses.

To reformulate the state-space into an ARMA(1,1) model, we can combine, the measurement equation with the state-transition equation as follows:

\[ r_{t+1}^e = \mu_{t+1} + \varepsilon_{r,t+1}, \]
\[ = (\rho(S_t)\mu_t + \varepsilon_{\mu,t+1}) + \varepsilon_{r,t+1} \]
\[ = \rho(S_t)r_t^e - \rho(S_t)\varepsilon_{r,t} + \varepsilon_{\mu,t+1} + \varepsilon_{r,t+1} \]

Since conditional on being in regime \( S_t \), \( \varepsilon_{\mu,t+1} \) and \( \varepsilon_{r,t+1} \) are normally distributed, their sum, \( \eta_{t+1} = \varepsilon_{\mu,t+1} + \varepsilon_{r,t+1} \), follows a normal distribution with zero mean and variance \( \sigma^2_{\eta}(S_t) = \sigma^2_{\varepsilon_r}(S_t) + \sigma^2_{\varepsilon_{\mu}}(S_t) + 2\sigma^2_{\varepsilon_{\mu}}(S_t) \). Hence, if we let \( \eta_{t+1} \sim N(0, \sigma^2_{\eta}(S_t)) \) and rescale the variance it follows:

\[ r_{t+1}^e = \rho(S_t)r_t^e - \rho(S_t)\varepsilon_{r,t} + \varepsilon_{\mu,t+1} + \varepsilon_{r,t+1} \]

\[ = \gamma(S_t)r_t^e + \psi(S_t)\eta_t + \eta_{t+1} \] (B.2)

which is a Markov-switching ARMA(1,1) process with AR coefficient \( \gamma(S_t) \), and MA and variance parameters equal to \( \psi(S_t) \) and \( \sigma^2_{\eta}(S_t) \), respectively.

\( \Sigma(S_t) \) is not identified. Let \( \theta(S_t) = [\rho(S_t), \text{vech}(\Sigma(S_t))]' \) be a vector that stacks all the parameters associated with the state-space representation for a given regime. From equation (B.2) we have
that for each $S_t$ the following set of conditions are satisfied

$$
\psi(S_t) = -\rho(S_t) \frac{\sigma_{\varepsilon_r}(S_t)}{\sqrt{\sigma_{\varepsilon_r}(S_t) + \sigma_{\varepsilon_\mu}(S_t) + 2\sigma_{\varepsilon_{\mu}}(S_t)}}
$$  \hspace{1cm} (B.3)

$$
\sigma^2_{\eta}(S_t) = \sigma^2_{\varepsilon_r}(S_t) + \sigma^2_{\varepsilon_\mu}(S_t) + 2\sigma^2_{\varepsilon_{\mu}}(S_t)
$$

Based on the ARMA(1,1) representation and given that $\gamma(S_t) = \rho(S_t)$, we see that at most 4 parameters are identified (i.e., $\psi(S_t)$ and $\sigma^2_{\varepsilon_r}(S_t)$ for $S_t \in \{0, 1\}$), but $\Sigma(S_t)$ in (B.1) has 6 parameters in total (i.e., $\sigma^2_{\varepsilon_r}(S_t)$, $\sigma^2_{\varepsilon_\mu}(S_t)$, and $\sigma^2_{\varepsilon_{\mu}}(S_t)$ for $S_t \in \{0, 1\}$). To show that the model is not identified, let $\delta > 0$ and specify $\sigma^2_{\varepsilon_{\mu,0}}(S_t) = \sigma^2_{\varepsilon_\mu}(S_t) + \delta$ and $\sigma^2_{\varepsilon_{\mu,0}}(S_t) = \sigma^2_{\varepsilon_{\mu}}(S_t) - \frac{1}{2}\delta$. The variance-covariance matrix associated with these parameters is given by

$$
\Sigma_0(S_{t+1}) = \begin{bmatrix}
\sigma^2_{\varepsilon_r}(S_t) & \sigma^2_{\varepsilon_{\mu,0}}(S_t) \\
\sigma^2_{\varepsilon_{\mu,0}}(S_t) & \sigma^2_{\varepsilon_{\mu,0}}(S_t)
\end{bmatrix} = \begin{bmatrix}
\sigma^2_{\varepsilon_r}(S_t) & \sigma^2_{\varepsilon_{\mu}}(S_t) - \frac{1}{2}\delta \\
\sigma^2_{\varepsilon_{\mu}}(S_t) - \frac{1}{2}\delta & \sigma^2_{\varepsilon_{\mu}}(S_t) + \delta
\end{bmatrix}
$$  \hspace{1cm} (B.4)

From equation (B.3) we see that $\theta_0(S_t) = [\rho(S_t), vec(h(S_t))]'$ delivers the same ARMA(1,1) model parameters $\psi(S_t)$ and $\sigma^2_{\eta}(S_t)$, but $\theta_0(S_t) \neq \theta(S_t)$ since $\delta > 0$. Hence, we conclude that the state-space model in (B.1) is not identified.

$\Sigma(S_t)$ is identified if restricted. In Section 2.2 I imposed two parameter restrictions. Here I show that these restrictions achieve identification. The first restriction was fixing the parameter $\sigma^2_r$ which denotes the steady state variance of $r^e$ conditional on $S_t = 0$:

$$
Var_{S_t}(r^e_{t+1}) = \sigma^2_{\varepsilon_r}(S_t) = \sigma^2_{\varepsilon_r}(1 + h \ S_t)
$$  \hspace{1cm} (B.5)

With respect to the second restriction, I set the correlation parameter between the unexpected return and shocks to the expected return to be equal across regimes (i.e., $\rho_{\varepsilon_\mu,r}(S_{t+1}) = \rho_{\varepsilon_\mu,r}$ for $S_t \in \{0, 1\}$).

To simplify notation define $\phi(S_t)$ as the population $R^2$ conditional on regime $S_t$ defined as $r^e_{t+1}$:

$$
\phi(S_t) = \frac{Var_{S_t}(\mu)}{Var_{S_t}(r^e_{t+1})} = \frac{1}{(1-\rho(S_t))^2} \sigma^2_{\varepsilon_\mu}(S_t)
$$  \hspace{1cm} (B.6)
and express

\[
\begin{align*}
\sigma^2_{\varepsilon \mu}(S_t) &= (1 - \rho(S_t)^2)\phi(S_t)\sigma_r^2(1 + h S_t) \\
\sigma^2_{\varepsilon \mu}(S_t) &= (1 - \phi(S_t))\sigma_r^2(1 + h S_t) \\
\sigma^2_{\varepsilon \mu}(S_t) &= \rho_{\varepsilon \mu, r} \sqrt{(1 - \rho(S_t)^2)\phi(S_{t+1})(1 - \phi(S_t))}\sigma_r^2(1 + h S_t)
\end{align*}
\]

(B.7)

where we now have only six parameters to estimate (i.e., \(\rho(S_t)^2\), \(\phi(S_t)\), \(h\) and \(\rho_{\varepsilon \mu, r}\) for \(S_t \in \{0, 1\}\)), which exactly equals the number of free parameters of the ARMA(1,1) representation. Using equations (B.3) and (B.7) and conditioning on \(S_t = 0\) we have

\[
\psi_0 = -\rho_0 \frac{\sqrt{1 - \phi_0}}{\sqrt{(1 - \rho_0^2)\phi_0 + (1 - \phi_0) + 2\rho_{\varepsilon \mu, r} \sqrt{(1 - \rho_0^2)\phi_0(1 - \phi_0)}}}
\]

(B.8)

\[
\sigma^2_{\eta, 0} = \left(1 - \rho_0^2\right)\phi_0 + (1 - \phi_0) + 2\rho_{\varepsilon \mu, r} \sqrt{(1 - \rho_0^2)\phi_0(1 - \phi_0)}\right)\sigma_r^2
\]

and \(\gamma_0 = \rho_0\). For fix \(\sigma^2_r\), (B.8) is a system of two non-linear equations where the two unknowns are \(\phi_0\) and \(\rho_{\varepsilon \mu, r}\). Similarly, conditioning on \(S_t = 1\) we have

\[
\psi_1 = -\rho_1 \frac{\sqrt{1 - \phi_1}}{\sqrt{(1 - \rho_1^2)\phi_1 + (1 - \phi_1) + 2\rho_{\varepsilon \mu, r} \sqrt{(1 - \rho_1^2)\phi_1(1 - \phi_1)}}}
\]

(B.9)

\[
\sigma^2_{\eta, 1} = \left(1 - \rho_1^2\right)\phi_1 + (1 - \phi_1) + 2\rho_{\varepsilon \mu, r} \sqrt{(1 - \rho_1^2)\phi_1(1 - \phi_1)}\right)\sigma_r^2(1 + h)
\]

and \(\gamma_1 = \rho_1\). Note that in the first line of (B.9), we can solve for \(\phi_1\), while in the second line we can solve for \(h\) since we already solved for \(\rho_{\varepsilon \mu, r}\) in (B.8). Provided that there exist a unique solution, we have proved the following theorem:

**Theorem.** Suppose that the ARMA(1,1) in (B.2) is identified and that a unique solution to (B.8) and (B.9) exists. Then the state-space model is identified if \(\Sigma(S_t)\) is restricted as in Section 2.2.